Unisex Tariffs in Insurance Markets
Equilibria and Social Welfare

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Unisex tariffs

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2004 EU Gender Derivative: Insurance companies are legally obliged to offer unisex tariffs, i.e. to offer the same contracts to both male and female clients, unless there is statistical evidence that identifies the client’s gender as a significant risk factor.

European Court of Justice: On March 1, 2011, the qualification in the unless-clause was ruled illegitimate. Thus, for all contracts entered on or after December 21, 2012, insurance companies will not be able to differentiate between the sexes; only unisex insurance policies will be available.

Charter of Fundamental Rights of the EU, Article 21: “Non-discrimination: 1. Any discrimination based on any ground such as sex, race, colour, ethnic or social origin, genetic features, language, religion or belief, political or any other opinion, membership of a national minority, property, birth, disability, age or sexual orientation shall be prohibited.”
In a first reaction, consumer organizations in Germany celebrated the judgment, assuming that unisex premia would roughly be the average of former premia for female and male individuals.

Insurance companies were quite unhappy and expected on average higher premia mainly arguing e.g. that pooling high- and low risk individuals we add additional randomness.

Both arguments underestimate adverse effects of mandatory unisex tariffs in free insurance markets:

Every individual can choose to buy less insurance or no insurance at all.
Our aim

Our aim is to provide

- a quantitative analysis of the associated insurance market equilibria
- and to investigate the welfare loss caused by regulatory adverse selection.
Model
Rationality

- Individuals have same wealth $x > 0$.
- Individuals have same utility function $u$.
- Example: CRRA – utility functions

$$u(x) = \frac{1}{1 - \rho} x^{1-\rho}, \quad \rho > 0.$$ 

$\rho = 1$ corresponds to logarithmic utility.

$\rho$ is the Arrow-Pratt Index of relative risk aversion.
Low- and high risk individuals

- Individuals are grouped into $\oplus$- and $\ominus$-agents.
- $\omega_{\oplus}$, $\omega_{\ominus}$ are the corresponding fractions, $\omega_{\oplus} + \omega_{\ominus} = 1$.
- Individuals of type $\oplus/\ominus$ face a risk $Z_{\oplus/\ominus}$ that is modeled as

$$P(Z_{\oplus/\ominus} = z_{\oplus/\ominus}) = \ell_{\oplus/\ominus}, \quad P(Z_{\oplus/\ominus} = 0) = 1 - \ell_{\oplus/\ominus}.$$ 

- We assume $|z_{\oplus}| \leq |z_{\ominus}|$, $\ell_{\oplus} \leq \ell_{\ominus}$, i.e. $\oplus$ “good” risks, $\ominus$ “bad” risks.

<table>
<thead>
<tr>
<th>insurance type</th>
<th>insured event</th>
<th>wealth $x &gt; 0$</th>
<th>loss $z_{\oplus/\ominus}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>accident/disability</td>
<td>injury</td>
<td>wealth of insured person</td>
<td>loss of earnings caused by disability</td>
</tr>
<tr>
<td>life</td>
<td>death of primary earner</td>
<td>household’s total wealth</td>
<td>loss of income</td>
</tr>
<tr>
<td>pension/endowment</td>
<td>longevity</td>
<td>total wealth for normal life span</td>
<td>shortfall in case of longevity</td>
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</table>
Premia and coverage (in a setting of price-competition)

- For premium $p_{\oplus/\ominus}$ the insurer covers the loss $z_{\oplus/\ominus}$.
- Individuals $\oplus/\ominus$ can buy coverage $\pi_{\oplus/\ominus}|z_{\oplus/\ominus}|$ for premium $\pi_{\oplus/\ominus}p_{\oplus/\ominus}$.
- For unisex tariffs, the insurance company offers only $p_{\oplus} = p_{\ominus} = p_{\odot}$.
- An insurance contract is feasible, if

$$E[\omega_{\oplus}\pi_{\oplus}(p_{\oplus} + Z_{\oplus}) + \omega_{\ominus}\pi_{\ominus}(p_{\ominus} + Z_{\ominus})] \geq 0.$$ 

- Optimization problem of individuals (insurance buyers) for given $p$:

$$\max_{\pi_{\oplus/\ominus} \geq 0} E\left[u(x - \pi_{\oplus/\ominus}p + (1 - \pi_{\oplus/\ominus})Z_{\oplus/\ominus})\right].$$

This yields for CRRA utility

$$\pi^*_{\oplus/\ominus}(p) = \max \left\{ \frac{x + z_{\oplus/\ominus} - \gamma^p_{\oplus/\ominus} x}{p + z_{\oplus/\ominus} - \gamma^p_{\oplus/\ominus} p}, 0 \right\}, \quad \gamma_{\oplus/\ominus} = -\frac{\ell_{\oplus/\ominus}}{1 - \ell_{\oplus/\ominus}} \frac{p + z_{\oplus/\ominus}}{p}.$$
Price competition
Scenarios for price competition

We consider two competition regimes

(M) monopolistic insurer
(C) perfect competition.

In regime (M), the insurer optimizes the expected profits.
In regime (C), the insurer prices by the net premium principle, i.e.

$$E[\omega_\oplus\pi_\oplus(p_\oplus + Z_\oplus) + \omega_\ominus\pi_\ominus(p_\ominus + Z_\ominus)] = 0.$$ 

In addition we consider two regulatory regimes

(E) mandatory equal tariffs (ban on discriminative policies),
(F) free contract design (including discriminative policies).

In regime (E), the insurer offers the same premium $p_\ominus$ to all clients.
In regime (F), the insurer is allowed to offer $p_\oplus (p_\ominus)$ to $\oplus(\ominus)$-agents.
Equilibrium with monopolistic insurance supply (M)

- The monopolistic insurer sets premia so as to extract maximum profits.
  - In regime (E) the premium \( \hat{p}_\circ \) is given by
    \[
    \hat{p}_\circ = \arg\max_{p_\circ} \{ \omega_\oplus \pi_\oplus^*(p_\circ)(p_\circ + \alpha_\oplus) + \omega_\ominus \pi_\ominus^*(p_\circ)(p_\circ + \alpha_\ominus) \}
    \]
    Then for \( \hat{\pi}_\oplus/\ominus := \pi_\oplus/\ominus^*(\hat{p}_\circ) \) the equilibrium premia would be \( \hat{\pi}_\oplus/\ominus \hat{p}_\circ \).
  - In (F) the premia \( \hat{p}_\oplus, \hat{p}_\ominus \) are given by
    \[
    (\hat{p}_\oplus, \hat{p}_\ominus) = \arg\max_{p_\oplus, p_\ominus} \{ \omega_\oplus \pi_\oplus^*(p_\oplus)(p_\oplus + \alpha_\oplus) + \omega_\ominus \pi_\ominus^*(p_\ominus)(p_\ominus + \alpha_\ominus) \}.
    \]
    Here, \( \alpha_\oplus := \ell_\oplus z_\oplus \) and \( \alpha_\ominus := \ell_\ominus z_\ominus \) are the expected losses.

- In general, no closed form solutions. Maximization is carried out numerically.
- In case of \( u = \log \) explicit solutions.
Equilibrium with monopolistic insurance supply (M)

Example for $\ell_\ominus = 5\%, \ z_\ominus = z_\ominus = -1, \ x_\ominus = x_\ominus = 2, \ \rho = 3$. 

Equilibrium insurance coverage (left) and equilibrium premia per coverage (right), as functions of $\ell_\ominus$ in market scenario (M)
Equilibrium with competitive insurance supply (C)

In the competitive scenario (C) the premium charged on any marketed contract is brought down so that its expected profit vanishes, i.e.

\[ E[\omega_\oplus \pi_\oplus (p_\oplus + Z_\oplus) + \omega_\ominus \pi_\ominus (p_\ominus + Z_\ominus)] = 0. \]

- In regime (F) it thus follows that
  \[ \hat{p}_\oplus = -\alpha_\oplus, \quad \hat{p}_\ominus = -\alpha_\ominus. \]

- and in regime (E) for given insurance demands \( \pi_\oplus \) and \( \pi_\ominus \), the premium is given by
  \[ p_\circ = \frac{\omega_\oplus \pi_\oplus}{\omega_\oplus \pi_\oplus + \omega_\ominus \pi_\ominus} (-\alpha_\oplus) + \frac{\omega_\ominus \pi_\ominus}{\omega_\oplus \pi_\oplus + \omega_\ominus \pi_\ominus} (-\alpha_\ominus). \]

In equilibrium we get \( \hat{p}_\circ \) and \( \hat{\pi}_{\oplus/\ominus} = \pi^*_\oplus/\ominus (\hat{p}_\circ) \).

- In general, no closed form solutions. Maximization is carried out numerically.

- In case of \( u = \log \) explicit solution.
Equilibrium with competitive insurance supply (C)

Example for $\ell_\oplus = 5\%, \; z_\ominus = z_\oplus = -1, \; x_\ominus = x_\oplus = 2, \; \rho = 3$.

Equilibrium insurance coverage (left) and equilibrium premia per coverage (right) as functions of $\ell_\ominus$ in competition regime (C).
Price-quantity-competition
Contrary to price competition we may consider price-quantity competition.

The insurance market is free and perfectly competitive.

Insurance companies offer several contracts $c = (p, \beta)$, each of which features a fixed premium $p$ and a fixed payment $\beta$ given damage, on a take-it-or-leave-it basis.

Linear scaling of premia and coverage is not possible.

Idea of Rothschild/Stiglitz (1976): by making price-quantity contingent offers $(p, \beta)$, insurers can induce individuals to purchase a contract specifically designed for their type in equilibrium (self-selection mechanism).

We use an analytic approach.
Equilibrium with price-quantity competition

- An **insurance market equilibrium** is a set of contracts $c = (p, \beta)$ satisfying
  1. No contract in the equilibrium set makes expected losses.
  2. There is no contract outside the equilibrium set such that, if it were offered, the expected utility of no agent would decrease while of at least one individual it would increase; and the contract would not make expected losses.

- If discriminative policies are allowed, then the equilibrium outcome is the same as in the competitive market (C) in the regulation regime (F).

- In regime (E) an equilibrium might not exist. But we know
  - There does not exist an equilibrium consisting of just a single contract, and it suffices to take two contracts $c_\oplus = (p_\oplus, \beta_\oplus), c_\ominus = (p_\ominus, \beta_\ominus)$ [RS76].
  - The equilibrium conditions for the pair $(c_\oplus, c_\ominus)$ can be reformulated as:
    1. Agents of type $\oplus/\ominus$ optimally choose contract $c_\oplus/\ominus$ (self-selection);
    2. $p_\oplus/\ominus - \ell_\oplus/\ominus \beta_\oplus/\ominus = 0$ (zero net profits);
    3. There is no contract $\tilde{c} = (\tilde{p}, \tilde{\beta})$ different from $c_\oplus$ and $c_\ominus$ that would not make expected losses and would improve utility of agents of type $\oplus$ or $\ominus$. 


Equilibrium with price-quantity competition

Example for $\ell_{\oplus} = 5\%$, $z_{\oplus} = z_{\ominus} = -1$, $x_{\ominus} = x_{\oplus} = 2$, $\rho = 3$.

Equilibrium insurance coverage (left) and equilibrium premia per coverage (right), as functions of $\ell_{\ominus}$ for price-quantity competition.
Welfare
Measuring welfare

- Individuals of both types maximize the certainty equivalent

\[
\max_{\pi/\Theta} u^{-1}\left(\mathbb{E}[u(x - \pi/\Theta p/\Theta + (1 - \pi/\Theta)Z/\Theta)]\right).
\]

- Equilibrium welfare thus becomes

\[
W_{\Theta/\Theta} = u^{-1}\left(\mathbb{E}[u(x - \hat{\pi}/\Theta \hat{\rho}/\Theta + (1 - \hat{\pi}/\Theta)Z/\Theta)]\right).
\]

- The social benefit, i.e. the monetary value of being part of the risk-sharing community, for an individual is

\[
W_{\Theta/\Theta} - W^0_{\Theta/\Theta}, \quad \text{where} \quad W^0_{\Theta/\Theta} = u^{-1}(\mathbb{E}[u(x + Z/\Theta)]).
\]

- The aggregate social benefit is thus given by

\[
W - W^0, \quad \text{where} \quad W = \omega_{\Theta} W_{\Theta} + \omega_{\Theta} W_{\Theta}, \quad W^0 = \omega_{\Theta} W^0_{\Theta} + \omega_{\Theta} W^0_{\Theta}.
\]

- Profits of insurance companies may be added depending on ownership.

- We compare to the no-insurance-regime (N).
Monopolistic insurance market

Example for $\ell_{\ominus} = 5\%, \ z_{\ominus} = z_{\oplus} = -1, \ x_{\ominus} = x_{\oplus} = 2, \ \rho = 3$. 

Welfare increase relative to regime (N) without (left) and with (right) insurance profits, as a function of $\ell_{\ominus}$ in market scenario (M).
Competitive insurance market

Example for \( \ell_\oplus = 5\% \), \( z_\ominus = z_\oplus = -1 \), \( x_\ominus = x_\oplus = 2 \), \( \rho = 3 \).

Welfare increase relative to regime (N) with pure price competition (left) and price-quantity competition (right), as a function of \( \ell_\ominus \).
Extensions
Influence of parameters

Comparative statics of our basic model can be analyzed numerically, e.g.

- Dependency on risk aversion $\rho$
- Dependency on coverage $z_{\Theta}$
- Dependency on population $\omega_{\Theta}$. 
Dependency on $\rho$ for price competition

Example for $\ell_{\oplus} = 5\%, \ell_{\ominus} = 20\%, z_{\ominus} = z_{\oplus} = -1, x_{\ominus} = x_{\oplus} = 2, \rho = 3$.

Equilibrium insurance coverage as a function of relative risk aversion $\rho$ in monopolistic (left) and price-competitive (right) market.
Dependency on $\rho$ for price-quantity competition

Example for $\ell_{\oplus} = 5\%$, $\ell_{\ominus} = 20\%$, $z_{\ominus} = z_{\oplus} = -1$, $x_{\ominus} = x_{\oplus} = 2$, $\rho = 3$.

Equilibrium insurance coverage as a function of $\rho$ under price-quantity competition (left); welfare increases relative to regime (N), as a function of relative risk aversion $\rho$ (right).
Conclusion
Some literature

- Akerlof (1970)
- Rothschild/Stiglitz (1976)
- Wilson (1977)
- Hoy (1982)
- Crocker and Snow (1986)
- McCarthy and Turner (1993)
- Doherty and Thistle (1996)
- Hoy and Polborn (2000)
- Polborn, Hoy ans Sadanand (2006)
- Hoy (2006)
- Riedel (2006)
- Kelly and Nielson (2006)
Conclusion

- We have formulated a basic stylized model of a generic insurance market.

- Our results show that mandatory unisex tariffs may result in significant welfare losses, unless the insurance market is governed by an outside monopolist and high- and low-risk individuals have similar characteristics.

- The basic intuition for this result is that for high-risk individuals, insurance is relatively more important than for low-risk individuals; hence the latter are faster to quit the insurance market than the former.

- Insurance allows to pool idiosyncratic risks on a macroeconomic scale and can achieve diversification and remove risk on the aggregate level. Unisex tariffs tend to impede risk-pooling and therefore typically reduce macroeconomic welfare.

- More realistic models may enforce, non-rationality may reduce these effects.