Aspects of Controlling Life Event Risks

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Based on joint works with Ninna Reitzel Jensen, Claus Munk,

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14th Scientific Day, DGVFM, Berlin, 30th April 2015

Uncertain Lifetime

$\swarrow$ **Humps** $\qquad$ $\nwarrow$ **Longevity**

Education $\longrightarrow$ **Human Investment** $\qquad$ Health decisions

Richard (1975): C-I-I under time-additive utility

$$\frac{dX(t)}{dt} = \underbrace{X(t)\,r}_{\text{capital gains}} + \underbrace{w(t)}_{\text{labor income}} - \underbrace{c(t)}_{\text{consumption}} - \underbrace{\widehat{\mu}(t)\,b(t)}_{\text{life insurance premium}}$$

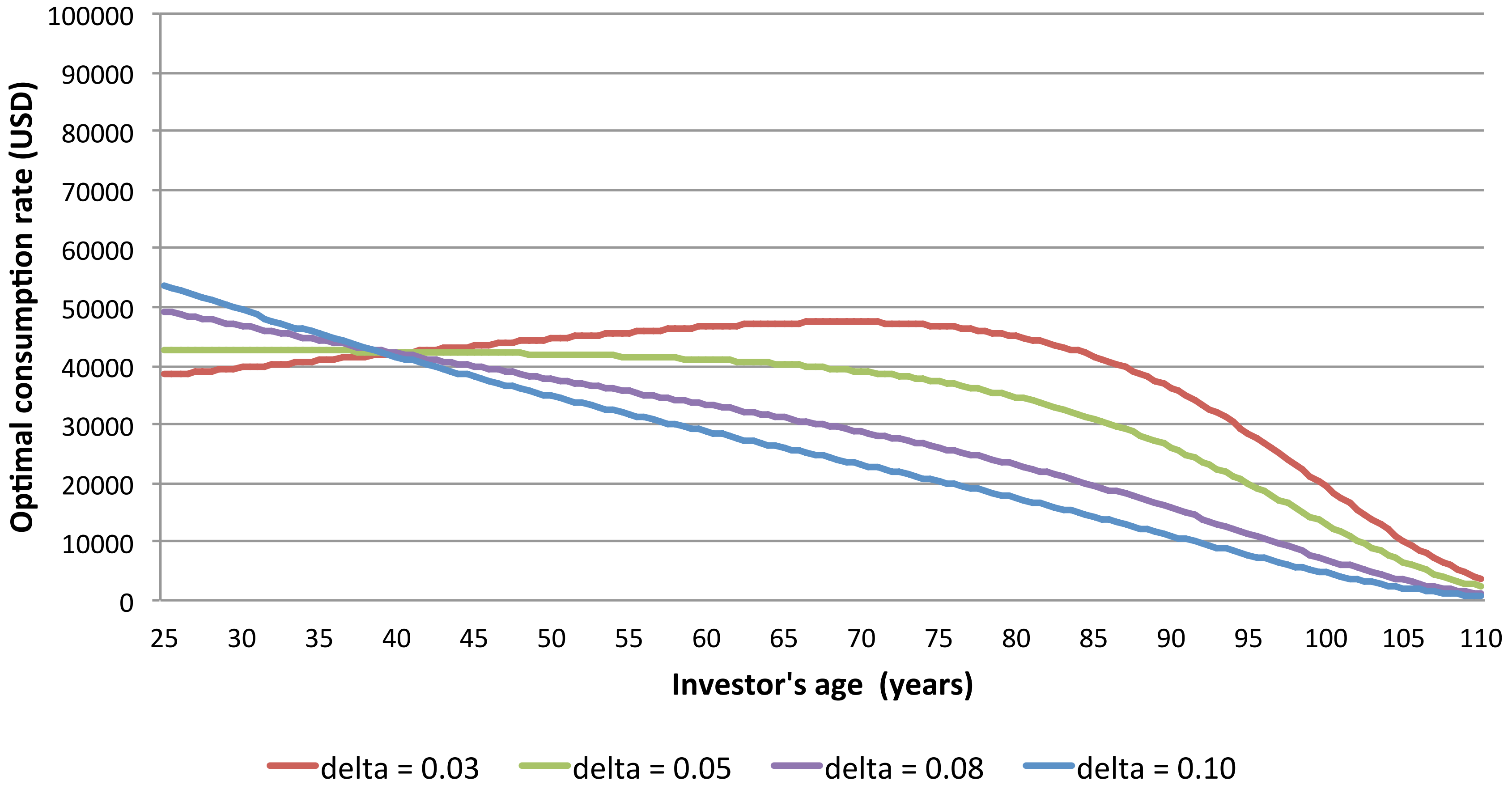
$$V(t,x) = \max_{c,b} E_{t,x}\left[\int_t^n \left(\underbrace{u(c(s))\,I(s)\,ds}_{\text{utility from consumption}} + \underbrace{u(X(s)+b(s))\,dN(s)}_{\text{utility from bequest}}\right)\right]$$

$$\begin{aligned} c^*(t,x) &= \underbrace{f(t)}_{\text{consumption-to-wealth ratio}}\left(x + \underbrace{h(t)}_{\text{human capital}}\right) \\ b^*(t,x) + x &= \underbrace{g(t)}_{\text{level of protection}}(x + h(t)) \end{aligned}$$

Jensen and Steffensen (2015): Personal finance and life insurance under separation of risk aversion and elasticity of substitution

$$V(t,x) = \int_t^n \underbrace{w}_{\text{EIS}} \left(v^{-1} \left(\begin{array}{c} \underbrace{v}_{\text{EBS}} \left(\underbrace{u^{-1}\left(E_{t,x}\left[u(c(s))\,I(s)\right]\right)}_{\text{certainty equivalent}} \right) \\ + \underbrace{v}_{\text{EBS}} \left(\underbrace{u^{-1}\left(E_{t,x}\left[u(X(s)+b(s))\,\frac{dN(s)}{ds}\right]\right)}_{\text{certainty equivalent}} \right) \end{array} \right) \right) ds$$

$$\begin{aligned} c^*(t,x) &= f(t)\,(x+h(t)) \\ b^*(t,x)+x &= g(t)\,(x+h(t)) \end{aligned}$$



100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
Optimal consumption rate (USD)
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
Investor's age (years)
delta = 0.03
delta = 0.05
delta = 0.08
delta = 0.10

Munk, Kraft, Seifried and Steffensen (2015) Consumption and Wage Humps in a Life-Cycle Model with Education

$$\begin{aligned}\frac{dX(t)}{dt} &= rX(t) + w(t) - c\,(t) - K\varepsilon\,(t)\\ \frac{dw(t)}{dt} &= \alpha\varepsilon\,(t) - \beta w\,(t)\end{aligned}$$

$$V\,(t,x) = \max_{c,\varepsilon} E_{t,x}\left[\int_t^n \frac{1}{1-\gamma}\left(c^{\kappa}\,(1-\varepsilon)^{1-\kappa}\right)^{1-\gamma} ds + \frac{1}{1-\gamma} X\,(n)^{1-\gamma}\right]$$

$$c^*(t,x,w) = f\,(t)\left(x + \underbrace{g\,(t)\,w + h\,(t)}_{\text{max human capital}}\right)$$

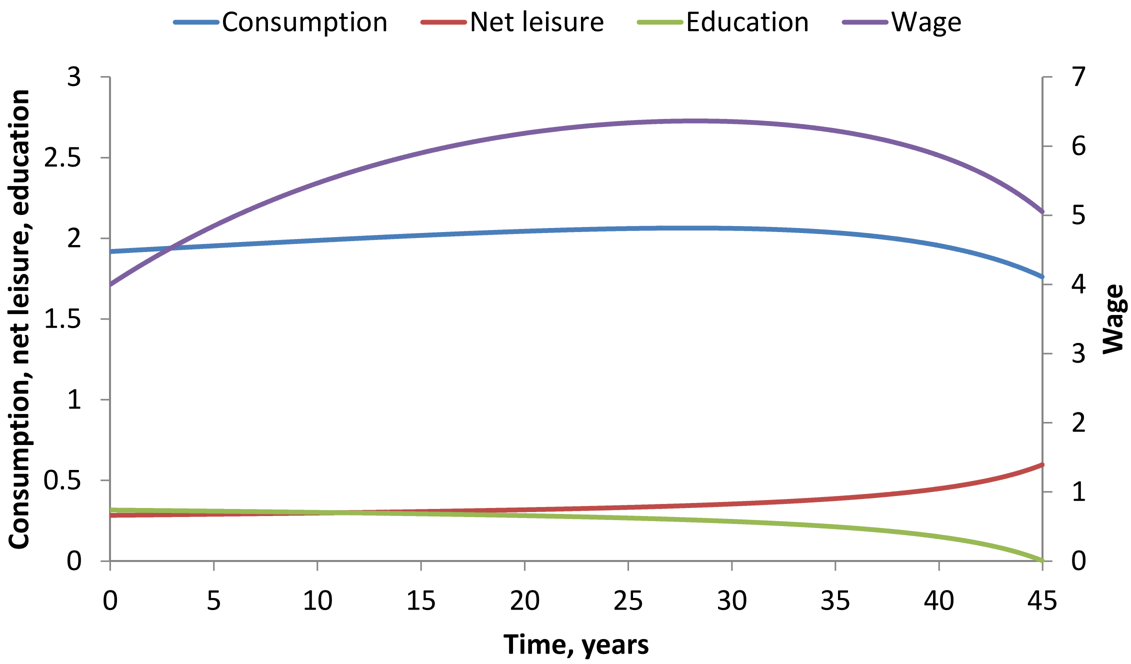


Figure 1: Life-cycle patterns for benchmark parameter values.
The figure depicts how the optimal consumption, education, and net leisure from Theorem 1 as well as the derived wage rate vary over the life of the agent. The benchmark parameter values listed in Table 1 are used. Note that consumption, net leisure, and education are read off the left axis, whereas the wage rate is read off the right axis.

With $\alpha = 1.5$ and $w = 4$, this implies that wages grow at a rate of around 12.4% per year less the 5% depreciation rate. The cost parameter $K = 1$ implies that the initially optimal educational effort costs 0.33 (corresponding to 3,300 USD) per year. The replacement rate of $\chi = 0.15$ means that, at retirement, the individual's income drops from 40% (equal to $1 - \ell$) to 15% of the hypothetical maximal annual income, but then stays at this level for the rest of his life. The initial wealth is set to 3 (corresponding to 30,000 USD).

Finally, we set the return r on savings to 4%. In our parsimonious deterministic setting, r should be interpreted as the return on a well-diversified portfolio. Hence a savings return of 4% appears realistic.

5.2 Life-cycle patterns

Figure 1 illustrates the life-cycle patterns generated by the solution presented in Theorem 1. The fraction of time spent on education (green curve; left axis) is initially around 0.33 and then gradually declines over time. The earlier in life the individual educates, the longer he can enjoy the positive effects of education on future wages. The flip-side of time spent on education is net leisure, which therefore increases over life (red curve; left axis).

Kraft, Seifried and Steffensen (work in progress). Optimal consumption and health decisions

$$\begin{aligned} \frac{dX(t)}{dt} &= X(t)\,r - k(\varepsilon(t)) - c(t) \\ \frac{d\mu(t)}{dt} &= \alpha(\varepsilon(t), \mu(t)) \end{aligned}$$

$$V(t,x) = \max_{c,l} E_{t,x}\left[\int_t^n u(c(s), \varepsilon(s))\, I(s)\, ds\right]$$

$$\begin{aligned} c^*(\mu, x) &= f_1(\mu)\, x \\ \varepsilon^*(\mu) &= f_2(\mu) \\ \frac{d\mu(t)}{dt} &= \alpha(\varepsilon^*(t), \mu(t)) \end{aligned}$$