# Asset Pricing under Uncertainty about Shock Propagation

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## Motivation

- **Time dimension of crisis**
  - economic contractions cluster
  - initial drops in aggregate consumption may increase likelihood of subsequent declines (World War I or II, Great Depression)
  - crisis modeled as bad state with high jump intensities
- **Multiple assets**
  - initial losses happen in one company/one sector/one country
  - initial losses may increase risk for whole economy
- **Uncertainty**: economic state is latent and has to be filtered
  - economic state is driven by rare events
  - some, but not all consumption shocks trigger bad state

  ⇒ high uncertainty about current state

## Our Contributions

**Contribution to theoretical asset pricing literature:**
Solution of a general equilibrium exchange economy with...

- **two Lucas trees**
  (Cochrane et al. (2008), Martin (2013))
- **jump-diffusion processes**
  (Barro (2006, 2009), Wachter (2013), Backus et al. (2011))
- **representative investor with recursive preferences**
  (Duffie/Epstein (1992), Benzoni et al. (2011))
- **contagious jumps / network effects**
  (Branger et al. (2013), Ait-Sahalia et al. (2012), Buraschi/Porchia (2012))
- **partial information about economic state**
  (filtering from jump-diffusion processes)
  (Branger et al. (2013), Croce et al. (2012), Pastor/Veronesi (2009))

## Our Contributions

- Equity risk premium
  - significant diffusion risk premia (due to uncertainty)
  - significant jump risk premia (due to risk of contagious jumps)
  - time dimension of contagion crucial
- Second moments of returns
  - excess volatility (due to incomplete information)
  - countercyclical volatilities of returns
  - countercyclical comovement of returns
- Predictability
  - consumption not predictable by price-dividend ratio
  - returns are predictable by price-dividend ratio
- Flat term structure of equity for maturities $\geq$ 5 years
- Heterogeneity of trees matters
  - toxic assets: higher risk premia, higher return volatilities
  - higher persistence of price-dividend ratios

## Consumption

- Endowment economy with **two Lucas trees**, total endowment is $C = C_A + C_B$
- Two economic states: **good/calm ($p_t = 1$) vs. bad/contagion ($p_t = 0$)**
- Dynamics of endowment $C_i$ depend on economic state

$$\frac{dC_{i,t}}{C_{i,t-}} = \mu_{i,t}dt + \sigma_i dW_{i,t} + L_i dN_{i,t}^{calm,calm} + L_i dN_{i,t}^{calm,cont} + L_i dN_{i,t}^{cont,cont}$$

drift and jump intensities are stochastic

$$\mu_{i,t} = p_t \mu_i^{calm} + (1 - p_t)\mu_i^{cont}$$

$$\lambda_{i,t}^{calm,calm} = p_t \lambda_i^{calm,calm} \qquad \lambda_{i,t}^{calm,cont} = p_t \lambda_i^{calm,cont}$$

$$\lambda_{i,t}^{cont,cont} = (1 - p_t)\lambda_i^{cont,cont}$$

- Special feature of our model: contagious shocks $dN_{i,t}^{calm,cont}$

  (immediate loss in one tree and simultaneous transition into the bad contagion state which affects **both trees**)

## Partial Information

- Major assumption: **economic state (i.e. $p_t$) is unobservable**
- Investor has to learn about state from observing $C_{A,t}$ and $C_{B,t}$
- $\widehat{p}_t$: estimated probability of being in the calm state
- Endowment processes under the investor's filtration $(\mathcal{G}_t)_t$:

$$\frac{dC_{i,t}}{C_{i,t-}} = \widehat{\mu}_{i,t}dt + \sigma_i d\widehat{W}_{i,t} + L_i d\widehat{N}_{i,t}$$

with estimated ("subjective") drift and jump intensity

$$\widehat{\mu}_{i,t} = \widehat{p}_t \mu_i^{calm} + (1 - \widehat{p}_t)\mu_i^{cont}$$
$$\widehat{\lambda}_{i,t} = \widehat{p}_t \left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont}\right) + (1 - \widehat{p}_t)\lambda_i^{cont,cont}$$

- $d\widehat{W}_{i,t}$ and $d\widehat{N}_{i,t}$: "observed" diffusion/jump processes

## Investor: Recursive Preferences

- Representative investor with **recursive preferences** maximizes

$$J_0 = E\left[\int_0^\infty f(C_t, J_t)dt\right]$$

with RRA $\gamma$, EIS $\psi$, time preference rate $\beta$, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}} < 1$

- Dividends are modeled as levered endowment $D_{i,t} = C_{i,t}^{\phi}$

$$\frac{dD_{i,t}}{D_{i,t-}} = \left(\phi\widehat{\mu}_{i,t} + \frac{1}{2}\phi(\phi-1)\sigma_i^2\right)dt + \phi\sigma_i d\widehat{W}_{i,t} + \left((1+L_i)^{\phi} - 1\right)d\widehat{N}_{i,t}$$

- **Equilibrium prices depend on two state variables**
  - perceived probability of being in the calm state $\widehat{p}$
  - relative size of the trees ($s_A = \frac{C_A}{C_A+C_B}$)

## Calibration

| | |
|---|---|
| $\mu_i^{calm}$ | **0.047** |
| $\mu_i^{cont}$ | **0.019** |
| $\sigma_i$ | **0.01** |
| $\rho$ | **0** |
| $L_i$ | **-0.06** |
| $\lambda_i^{calm,calm}$ | **0.125** |
| $\lambda_i^{calm,cont}$ | **0.125** |
| $\lambda_i^{cont,cont}$ | **0.8** |
| $\lambda^{cont,calm}$ | **1** |

- Unconditional expected annual consumption growth: $\approx 1.9\%$
- Stationary probabilities of the two states: 80% vs. 20%
- Both trees have identical parameters
- Leverage parameter $\phi = 2.5$
- Preference parameters: $\gamma = 10$, $\psi = 2$, $\beta = 0.039$

## Asset Pricing Moments

- Annualized moments from Monte-Carlo simulations

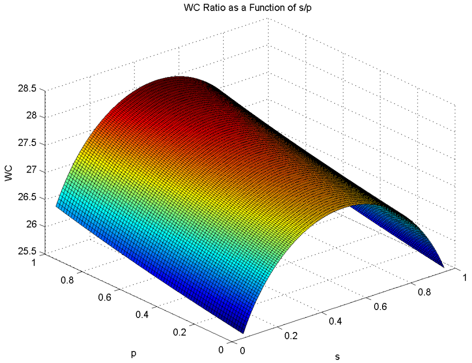
| **Moment** | $E[\Delta c]$ | $\sigma(\Delta c)$ | $AC1(\Delta c)$ | $E[\Delta d]$ | $\sigma(\Delta d)$ | $AC1(\Delta d)$ |
|---|---|---|---|---|---|---|
| **Data** | 0.019 | 0.013 | 0.42 | 0.022 | 0.066 | 0.25 |
| **Model** | 0.019 | 0.028 | 0.21 | 0.050 | 0.073 | 0.20 |

| **Moment** | $E[wc]$ | $\sigma[wc]$ | $AC1[wc]$ | $E[pd]$ | $\sigma[pd]$ | $AC1[pd]$ |
|---|---|---|---|---|---|---|
| **Data** | 4.63 | 0.247 | 0.86 | 3.38 | 0.423 | 0.88 |
| **Model** | 3.33 | 0.010 | 0.48 | 3.09 | 0.046 | 0.59 |

| **Moment** | $E[r_{Aggr} - r_f]$ | $\sigma(r_{Aggr} - r_f)$ | $AC1(r_{Aggr} - r_f)$ | $E[corr(r_A, r_B)]$ |
|---|---|---|---|---|
| **Data** | 0.069 | 0.174 | -0.01 | 0.72 |
| **Model** | 0.058 | 0.111 | -0.03 | 0.59 |

| **Moment** | $E[r_f]$ | $\sigma(r_f)$ |
|---|---|---|
| **Data** | 0.013 | 0.027 |
| **Model** | 0.035 | 0.013 |

## Wealth-Consumption Ratio

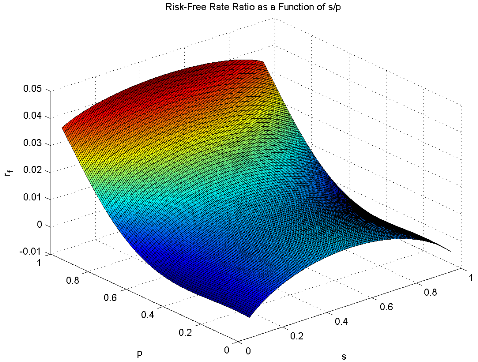



Independent variables

- $\widehat{p}$: perceived probability of being in the calm state
- $s_A = \frac{C_A}{C_A + C_B}$: relative size of the tree $A$

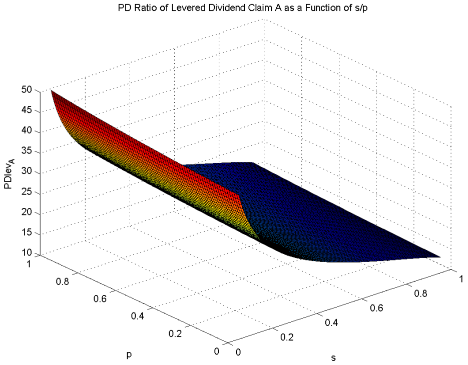
- Concave in $s$ ($\rightarrow$ aggregate consumption risk)
- Monotonously decreasing in $\widehat{p}$ ($\rightarrow$ state of the economy)
- Dependence of wealth-consumption-ratio on $s$ and $\widehat{p}$ drives risk premia for state variables

## Risk-Free Rate




- Risk-free rate is concave in $s$ ($\rightarrow$ precautionary savings)
- Risk-free rate is convex and increasing in $\widehat{p}$ ($\rightarrow$ precautionary savings and impact of $\widehat{p}$ on expected growth rate)

## Price-Dividend Ratio of Levered Claim $A$

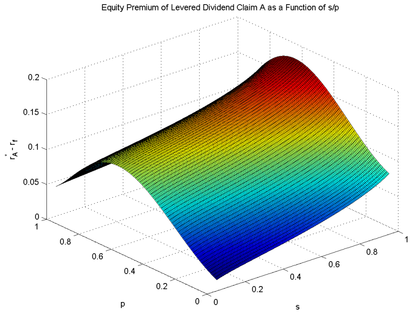



- PD-ratio monotonically decreasing in the share of asset $A$ (larger trees are less valuable, see also Cochrane et al. (2008))
- PD-ratio around 10 % smaller for $\widehat{p} = 0$ (contagion state)
- Impact of $s$ is larger than impact of $\widehat{p}$

## Risk Premia: Partial versus Full Information

Effects of **partial information** about contagion risk

1. premium for (deterministic) jump size: smaller
2. additional premium for diffusive $\widehat{p}$ risk, much larger than premium for diffusive $s$-risk
   $\Rightarrow$ large equity premium when uncertainty is high ($\hat{p} \approx 0.5$)

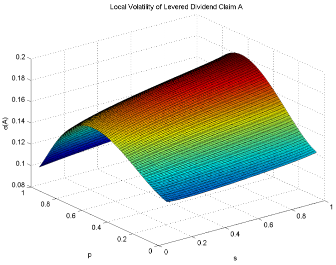


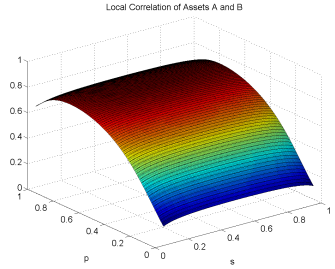

- Uncertainty about contagion has large impact on risk premia

$\Rightarrow$ Empirical risk premia can be matched with small jump sizes

($\rightarrow$ see also Backus, Chernov, Martin (2011))

## Local Correlations and Volatilities




- Large excess return volatility
- Large excess and procyclical correlations between asset returns
- **But:** ample empirical evidence for countercyclical correlations/volatilities

⇒ **Correlations and volatilities of monthly returns**

- simulate model and compute monthly returns
- compute return correlations/volatilities over 1, 2, 3, 4 years
- integrate $\widehat{p}$ paths over the same 1, 2, 3, 4 years
- regress correlations/volatilities on integrated $\widehat{p}$

## Correlations of Monthly Returns

Results (overlapping regressions)

| | Horizon (in years) | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| $\beta$ | -0.38 | -0.30 | -0.23 | -0.18 |
| $R^2$ | 0.09 | 0.07 | 0.06 | 0.06 |
| $E[\widehat{p}]$ | 0.80 | 0.80 | 0.80 | 0.80 |
| $\sigma(\widehat{p})$ | 0.31 | 0.27 | 0.24 | 0.21 |
| $E[corr(r_A^*, r_B)]$ | 0.47 | 0.54 | 0.58 | 0.60 |
| $\sigma(corr(r_A^*, r_B))$ | 0.39 | 0.31 | 0.28 | 0.22 |

⇒ Countercyclical correlations of monthly returns (as in the data)

- Robustness checks: weekly returns, non-overlapping data

## Volatilities of Monthly Returns

Results (overlapping regressions)

| | Horizon (in years) | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| $\beta$ | -0.14 | -0.15 | -0.16 | -0.16 |
| $R^2$ | 0.24 | 0.30 | 0.33 | 0.34 |
| $E[\widehat{p}]$ | 0.80 | 0.80 | 0.80 | 0.80 |
| $\sigma(\widehat{p})$ | 0.31 | 0.27 | 0.24 | 0.21 |
| $E[\sigma(r_A)]$ | 0.09 | 0.10 | 0.11 | 0.11 |
| $\sigma(\sigma(r_A))$ | 0.09 | 0.07 | 0.06 | 0.05 |

$\Rightarrow$ Countercyclical volatilities of monthly returns (as in the data)

- Robustness check: non-overlapping data

## Predictability of returns

| | Horizon (in years) | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 4 | 6 | 8 | 10 |
| | Panel A: Our Model, sample length 65 years | | | | | |
| $\beta$ | -0.19 | -0.27 | -0.40 | -0.52 | -0.62 | -0.72 |
| $R^2$ | 0.03 | 0.03 | 0.04 | 0.05 | 0.05 | 0.06 |
| | Panel B: Wachter (2013) | | | | | |
| $\beta$ | -0.11 | -0.22 | -0.40 | -0.56 | -0.69 | -0.82 |
| $R^2$ | 0.04 | 0.08 | 0.15 | 0.20 | 0.23 | 0.26 |
| | Panel C: Data | | | | | |
| $\beta$ | -0.13 | -0.23 | -0.33 | -0.48 | -0.64 | -0.86 |
| $R^2$ | 0.09 | 0.17 | 0.23 | 0.30 | 0.38 | 0.43 |

$\Rightarrow$ Returns are predictable by the price-dividend ratio (as in the data)

## Predictive Regressions

# Predictability of consumption growth

| | Horizon (in years) | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 4 | 6 | 8 | 10 |
| | Panel A: Our Model, sample length 65 years | | | | | |
| $\beta$ | 0.02 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 |
| $R^2$ | 0.11 | 0.08 | 0.05 | 0.04 | 0.04 | 0.04 |
| | Panel B: Wachter (2013) | | | | | |
| $\beta$ | 0.02 | 0.04 | 0.07 | 0.10 | 0.12 | 0.13 |
| $R^2$ | 0.01 | 0.02 | 0.04 | 0.05 | 0.06 | 0.06 |
| | Panel C: Data | | | | | |
| $\beta$ | -0.001 | - 0.006 | -0.009 | -0.011 | -0.016 | -0.014 |
| $R^2$ | 0.0006 | 0.0137 | 0.0164 | 0.0180 | 0.0268 | 0.0162 |

$\Rightarrow$ Consumption growth is not predictable by price-dividend ratio (as in the data)

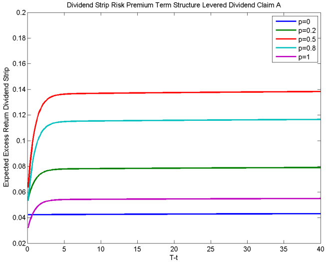
$\Rightarrow$ Dividend growth is also not predictable by price-dividend ratio

## Term Structure of Equity

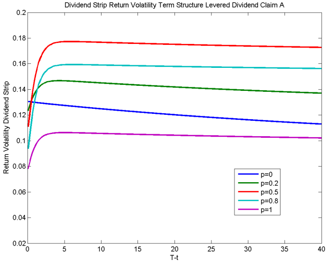
- Empirical findings: term structure of equity seems to be downward-sloping or flat
- Long-run risk models: typically produce an upward-sloping term structure
- Croce et al. (2012): **unobservable long-run risk** can generate downward-sloping equity term structures
- Intuition
  - short-run (consumption) risk and long-run (consumption growth) risk can no longer be perfectly disentangled
  - perceived short-run risk too high
  - perceived long-run risk too low
- Similarly, in our economy
  - "long-run risk" $\rightarrow$ contagious shock
  - "short-run risk" $\rightarrow$ normal jumps
  - investor cannot distinguish between different types of jumps

## Term Structure of Equity

### Risk Premia




### Return Volatilities




- Term structure of risk premia less strongly upward-sloping than in usual long-run risk models
- Term structure of volatilities hump-shaped or even downward sloping

## Cross-Sectional Results: Toxic vs. Non-Toxic Assets

Assets differ in propensity to trigger contagion

| | Benchmark Economy | Toxic vs. Non-Toxic Assets |
|---|---|---|
| $\lambda_A^{calm,calm}$ | **0.125** | **0** |
| $\lambda_A^{calm,cont}$ | **0.125** | **0.25** |
| $\lambda_B^{calm,calm}$ | **0.125** | **0.25** |
| $\lambda_B^{calm,cont}$ | **0.125** | **0** |
| $\lambda_i^{cont,cont}$ | **0.8** | **0.8** |
| $\lambda^{cont,calm}$ | **1** | **1** |

- Asset A: **toxic asset**
- Asset B: **non-toxic asset**
- Local distributions of both assets: no change
- Preferences and leverage: no change

## Asset Pricing Moments: Toxic vs. Non-Toxic Assets

- Annualized moments from Monte-Carlo simulations

| **Moment** | $E[\Delta c]$ | $\sigma(\Delta c)$ | $AC1(\Delta c)$ | $E[\Delta d]$ | $\sigma(\Delta d)$ | $AC1(\Delta d)$ |
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| **Data** | 0.019 | 0.013 | 0.42 | 0.022 | 0.066 | 0.25 |
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| **Data** | 4.63 | 0.247 | 0.86 | 3.38 | 0.423 | 0.88 |
| **Model** | 3.33 | 0.011 | 0.51 | 3.11 | 0.050 | 0.65 |

| **Moment** | $E[r_A - r_f]$ | $\sigma(r_A - r_f)$ | $AC1(r_A - r_f)$ | $E[corr(r_A, r_B)]$ |
|---|---|---|---|---|
| **Data** | - | - | -0.01 | 0.72 |
| **Model** | 0.061 | 0.129 | -0.02 | 0.61 |

| **Moment** | $E[r_B - r_f]$ | $\sigma(r_B - r_f)$ | $AC1(r_B - r_f)$ | $E[r_f]$ | $\sigma(r_f)$ |
|---|---|---|---|---|---|
| **Data** | - | - | -0.01 | 0.013 | 0.027 |
| **Model** | 0.053 | 0.115 | -0.04 | 0.034 | 0.012 |

## Conclusion

- Equilibrium with two trees, two regimes, jumps, recursive utility and partial information about contagion risk
- **Key features:**
  - regime switch coupled to jump in endowment
  - investor has to learn about the economic state
- **Results**
  - Equity risk premium not mainly due to jump risk, but also due to diffusion risk (higher uncertainty due to partial information)
  - Volatilities and comovement of returns: countercyclical
  - Predictability: consumption is not predictable by pd-ratio, returns are predictable
  - Flat term structure of equity for maturities $\geq$ 5 years
  - Heterogeneity of trees matters: increases persistence of returns, toxic assets have higher risk premium and higher volatility