Old and New Problems in Insurance Contract Design

Enrico Biffis
Imperial College London

DGVFM Scientific Day 2013
Berlin
April 26, 2013
OUTLINE

1. CSA design
2. A Model
3. Optimal Collateral Rules
4. Longevity swaps
5. Dynamic adverse selection
6. Conclusion
OVERVIEW

Regulation of OTC markets

- Big Bang Protocol, Dodd-Frank, EMIR, etc.
- move to centrally cleared transactions for (some) OTC derivatives
- collateralization of non-cleared derivatives
Regulation of OTC markets

- Big Bang Protocol, Dodd-Frank, EMIR, etc.
- move to centrally cleared transactions for (some) OTC derivatives
- collateralization of non-cleared derivatives

### Table 3: Non-centrally cleared derivative activity before and after central clearing takes effect

<table>
<thead>
<tr>
<th></th>
<th>Total gross notional outstanding amount (EUR million)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Foreign exchange</td>
</tr>
<tr>
<td>Before</td>
<td>54,958,056</td>
</tr>
<tr>
<td>After</td>
<td>47,863,156</td>
</tr>
<tr>
<td>% Reduction</td>
<td>13%</td>
</tr>
</tbody>
</table>

Note: The data above reflect the notional amount of non-centrally cleared derivative activity that will remain after central clearing mandates take effect (future portfolio). Each cell represents the simple sum of non-centrally cleared derivative notional amounts for each QIS respondent within each asset class and jurisdiction.

Source: BIS (2013)
OVERVIEW

Regulation of OTC markets

- Big Bang Protocol, Dodd-Frank, EMIR, etc.
- move to centrally cleared transactions for (some) OTC derivatives
- collateralization of non-cleared derivatives

Counterparty risk mitigation

- multi-curve valuation (OIS, EUREPO, etc.)
- Credit Support Annex (CSA), ISDA rules
OVERVIEW

Regulation of OTC markets
- Big Bang Protocol, Dodd-Frank, EMIR, etc.
- move to centrally cleared transactions for (some) OTC derivatives
- collateralization of non-cleared derivatives

Counterparty risk mitigation
- multi-curve valuation (OIS, EUREPO, etc.)
- Credit Support Annex (CSA), ISDA rules

Bilateral default risk in insurance
- bespoke longevity swaps
- hedging programmes and collateral management for UL/VA business
- counterparty risk relevant in retrocession and catastrophe insurance vehicles
**Bilateral default risk**

- Duffie/Huang (1996): essentially Cox setting, deflation by regime switching spread above risk-free rate (no collateral, exogenous recovery rates)
- Johannes/Sundaresan (2007): market-implied collateral cost in IRS markets
- Brigo/al. (2007+), Crepey (2011+), Hull and White (2010+), etc.: CVA/DVA/FVA
- Biffis/al. (2011), Brigo/al. (2012): **CSA pricing** and funding costs, but exogenous
**LITERATURE**

**Bilateral** default risk

- Duffie/Huang (1996): essentially Cox setting, deflation by regime switching spread above risk-free rate (no collateral, exogenous recovery rates)
- Johannes/Sundaresan (2007): market-implied collateral cost in IRS markets
- Brigo/al. (2007+), Crepey (2011+), Hull and White (2010+), etc.: CVA/DVA/FVA
- Biffis/al. (2011), Brigo/al. (2012): **CSA pricing** and funding costs, but **exogenous**

**Collateralization of non-cleared derivatives**

- Industry concerns about cost/scarcity of collateral: margin lending, liquidity swaps.
LITERATURE

Bilateral default risk

- Duffie/Huang (1996): essentially Cox setting, deflation by regime switching spread above risk-free rate (no collateral, exogenous recovery rates)
- Johannes/Sundaresan (2007): market-implied collateral cost in IRS markets
- Brigo/al. (2007+), Crepey (2011+), Hull and White (2010+), etc.: CVA/DVA/FVA
- Biffis/al. (2011), Brigo/al. (2012): CSA pricing and funding costs, but exogenous

Collateralization of non-cleared derivatives

- Industry concerns about cost/scarcity of collateral: margin lending, liquidity swaps.

My focus

- simple framework for CSA design and pricing
- cost of posting collateral: default penalties, funding costs, market incompleteness, fungibility of collateral (rehypothecation vs. segregation), close-out conventions
- potential impact of Dodd-Frank/EMIR provisions
Agent $i \in \{A, B\}$, endowed with wealth $w^i_0$, CARA utility.
Agent \( i \in \{A, B\} \), endowed with wealth \( w_0^i \), CARA utility

**Tradeable assets**

- money market account yielding \( r > 0 \)
- risky asset \( dS_t = S_t \left( \mu dt + \sigma S_t dB_t^{(1)} \right) \)
Agent $i \in \{A, B\}$, endowed with wealth $w^i_0$, CARA utility

Tradeable assets
- money market account yielding $r > 0$
- risky asset $dS_t = S_t \left( \mu dt + \sigma_S dB_t^{(1)} \right)$

Trading account dynamics ($i \in \{A, B\}$)

$$dW_t^i = W_t^i r dt + \pi_t^i \left( (\mu - r) dt + \sigma_S dB_t^{(1)} \right)$$
Setup

Agent $i \in \{A, B\}$, endowed with wealth $w_0^i$, CARA utility

Tradeable assets

- money market account yielding $r > 0$
- risky asset $dS_t = S_t \left( \mu dt + \sigma S dB_t^{(1)} \right)$

Trading account dynamics ($i \in \{A, B\}$)

$$dW_t^i = W_t^i r dt + \pi_t^i \left( (\mu - r) dt + \sigma S dB_t^{(1)} \right)$$

A has exposure $-Z_T$ at $T > 0$, B has exposure $+Z_T$, with

$$dZ_t = \sigma_Z dB_t^{(2)}, \quad Z_0 = 0$$

- $Z$ illiquid: agent (say) A has terminal wealth $W_T^A - Z_T$
- $Z$ unspanned: $B^{(1)} \perp B^{(2)}$
SETUP

Agent $i \in \{A, B\}$, endowed with wealth $w_0^i$, CARA utility

Tradeable assets

- money market account yielding $r > 0$
- risky asset $dS_t = S_t \left( \mu dt + \sigma_S dB_t^{(1)} \right)$

Trading account dynamics ($i \in \{A, B\}$)

$$dW^i_t = W^i_t r dt + \pi^i_t \left( (\mu - r) dt + \sigma_S dB_t^{(1)} \right)$$

A has exposure $-Z_T$ at $T > 0$, B has exposure $+Z_T$, with

$$dZ_t = \sigma_Z dB_t^{(2)}, \quad Z_0 = 0$$

- $Z$ illiquid: agent (say) A has terminal wealth $W^A_T - Z_T$
- $Z$ unspanned: $B^{(1)} \perp B^{(2)}$
- agents can enter a forward agreement on $k$ units of $Z_T$
Agent \( i \in \{ A, B \} \), endowed with wealth \( w_0^i \), CARA utility, default intensity \( \lambda > 0 \).

**Tradeable assets** *(only accessible in non-default states; Alvarez/Jermann, 2000)*
- money market account yielding \( r > 0 \)
- risky asset \( dS_t = S_t \left( \mu dt + \sigma_S dB_t^{(1)} \right) \)

**Trading account dynamics** *(\( i \in \{ A, B \} \))*

\[
dW_t^i = W_t^i rd\tau + \pi_t^i \left( (\mu - r)d\tau + \sigma_S dB_t^{(1)} \right)
\]

A has exposure \(-Z_T\) at \( T > 0 \), B has exposure \(+Z_T\), with

\[
dZ_t = \sigma_Z dB_t^{(2)}, \quad Z_0 = 0
\]

- \( Z \) illiquid: agent (say) A has terminal wealth \( W_T^A - Z_T \)
- \( Z \) unspanned: \( B^{(1)} \perp B^{(2)} \)
- agents can enter a forward agreement on \( k \) units of \( Z_T \), but are exposed to counterparty risk
PROBLEMS

OTC instrument

- bilateral transaction to exchange $kZ_T$ at $T > 0$
- focus on A for convenience; symmetry between A’s and B’s views

**Problem 1** (no counterparty risk)

\[
\begin{align*}
\sup_{(k, \pi) \in \mathbb{R} \times A_\pi} & \quad U \left( W_T^A - (1 - k)Z_T \right) \\
\text{s.t.} & \quad dW_t^A = W_t^A r dt + \pi_t \left( (\mu - r) dt + \sigma S dB_t^{(1)} \right) \\
& \quad dZ_t = \sigma_Z dB_t^{(2)}
\end{align*}
\]
PROBLEMS

OTC instrument

- bilateral transaction to exchange $kZ_T$ at $T > 0$
- focus on $A$ for convenience; symmetry until $\tau := \tau^A \land \tau^B$ ($N_t := 1_{\tau \leq t}$)

Problem 2 (counterparty risk, Zero CSA)

$$\sup_{(k,\pi)\in \mathbb{R} \times \mathcal{A}_\pi} U \left(W^A_T - (1 - k1_{\tau > T})Z_T\right)$$

\[
\begin{align*}
\text{st.} \quad & dW^A_t = N^A_t W^A_t r dt + (1 - N^A_t) W^A_t r dt \\
& \quad + (1 - N^A_t) W^A_t \pi_t \left((\mu - r)dt + \sigma_S dB^{(1)}_t\right) \\
& \quad + (1 - N^A_t) \left( (R^A_t)^+ dN^A_t - (R^A_t)^- dN^B_t \right) \\
& dZ_t = \sigma_Z dB^{(2)}_t
\end{align*}
\]

with $\tau^i$ default time of agent $i$ and $N^i_t := 1_{\tau^i \leq t}$ ($i \in \{A, B\}$).
PROBLEMS

OTC instrument

- bilateral transaction to exchange $kZ_T$ at $T > 0$
- focus on A for convenience; symmetry until $\tau := \tau^A \wedge \tau^B$ ($N_t := 1_{\tau \leq t}$)

**Problem 3** (counterparty risk, general CSA)

$$
\begin{align*}
&\sup_{(k,C^A,\pi) \in \mathbb{R} \times A_c \times A} \left\{ U \left( W^A_T - (1 - k1_{T > T})Z_T - 1_{T > T}C^A_T \right) \\
&\quad \text{s.t.} \right. \\
&\quad \text{where} \\
&\quad dW_t^A = N_t^A W_t^A r dt + (1 - N_t^{A_\tau}) W_t^A r dt \\
&\quad \quad + (1 - N_t^{A_\tau}) W_t^A \pi_t \left( (\mu - r) dt + \sigma_S d B_t^{(1)} \right) \\
&\quad \quad + (1 - N_t^{A_\tau}) \left[ dC_t^A - r C_t^A dt \right. \\
&\quad \quad \left. + \left( (R_t^A)^+ - (C_t^A)^+ \right) d N_t^A \\
&\quad \quad \left. + \left( (R_t^A)^- - (C_t^A)^- \right) d N_t^B \right] \\
&\quad \quad dZ_t = \sigma Z d B_t^{(2)}
\end{align*}
$$

- solve using symmetry on $[0, \tau)$ ...
OUTLINE

1. CSA design
2. A Model
3. Optimal Collateral Rules
4. Longevity swaps
5. Dynamic adverse selection
6. Conclusion
Agents A solves the problem

\[
\max_{(k, C^A, \pi) \in \mathbb{R} \times \mathcal{A}_c \times \mathcal{A}_\pi} U \left( W^A_T - (1 - k 1_{\tau > T}) Z_T - 1_{\tau > T} C^A_T \right)
\]

subject to her budget constraint (and agent B solving the symmetric problem).

- **Close-out convention**: risk-neutral, default-free replacement cost

\[
R^A_t(k) = E_t \left[ e^{-r(T-t)} (k Z_T) \right] = k e^{-r(T-t)} Z_t
\]

- **Admissible collateral rules**

\[
\mathcal{A}_c = \left\{ (C^A_t)_{t \in [0, \tau]} : C^A_t = c^A(t) R^A_t(k), c^A \in C^1([0, T]; \mathbb{R}_+), c^A = -c^B \right\}
\]
## TRADE-OFFS AND RESULTS

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Instrument</th>
<th>Insurance</th>
<th>Retention</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-Z$</td>
<td>forward</td>
<td>$k$</td>
<td>$1 - k$</td>
<td>$k^* = 1$</td>
</tr>
</tbody>
</table>

- $W \perp Z$
- zero forward price makes both parties equally better off
- full insurance optimal
## TRADE-OFFS AND RESULTS

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Instrument</th>
<th>Insurance</th>
<th>Retention</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-Z$</td>
<td>forward</td>
<td>$k$</td>
<td>$1 - k$</td>
<td>$k^* = 1$</td>
</tr>
<tr>
<td>$-Z, \tau$</td>
<td>forward</td>
<td>$k$</td>
<td>$1 - k$</td>
<td>$k^* &gt; 1$</td>
</tr>
<tr>
<td>ISDA CSA</td>
<td></td>
<td>$c$</td>
<td>$1 - c$</td>
<td>$c^* &lt; 1$</td>
</tr>
</tbody>
</table>

- $W$ and $(Z, \tau)$ dependent
- via default penalty ($\tau$), via collateral flows ($Z$) and close-out ($Z, \tau$)
- posting collateral is costly (collateral is “defaulter’s pay”)
- insure higher recovery at $T$ with larger position $k$
- underinsurance w.r.to $\tau$, overinsurance w.r.to $Z$
OPTIMAL DETERMINISTIC CSA \((c^* < 1)\)

Parameters: \((\gamma, \lambda, \sigma_Z, r, \mu, \sigma_S, W_0) = (0.2, 0.05, 0.2, 0.03, 0.06, 0.2, 0)\); \(k^*, T = 1, 2, \ldots, 10\).
## TRADE-OFFS AND RESULTS

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Instrument</th>
<th>Insurance</th>
<th>Retention</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-Z$</td>
<td>forward</td>
<td>$k$</td>
<td>$1 - k$</td>
<td>$k^* = 1$</td>
</tr>
<tr>
<td>$-Z, \tau$</td>
<td>forward</td>
<td>$k$</td>
<td>$1 - k$</td>
<td>$k^* &gt; 1$</td>
</tr>
<tr>
<td>ISDA CSA</td>
<td></td>
<td>$c$</td>
<td>$1 - c$</td>
<td>$c^* &lt; 1$</td>
</tr>
<tr>
<td>$-Z, \tau$</td>
<td>forward</td>
<td>$k$</td>
<td>$1 - k$</td>
<td>$k^* &lt; 1$</td>
</tr>
<tr>
<td>EMIR/Dodd-Frank</td>
<td></td>
<td>$c = 1$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\hat{c}^i = \hat{c} + c^i$</td>
<td></td>
<td>$\hat{c}$</td>
<td>tail risk</td>
<td></td>
</tr>
</tbody>
</table>

- as before, $W$ and $(Z, \tau)$ dependent
- two-way Initial Margin (IM), $\hat{c}$: 99% VaR over 10-day horizon (if daily collateral posting)
- variation margin: full collateralization, 100% of MTM
- $\tau$ still matters despite full collateralization
- costly collateralization forces underinsurance w.r.to $Z$
THE COST OF COUNTERPARTY RISK

The graph shows the expected utility as a function of horizon $T$. The line indicates no default risk. Parameters: $\lambda = 0$, otherwise baseline parameters.
THE COST OF COUNTERPARTY RISK

Parameters: $c = 0$, otherwise baseline parameters.
Baseline parameters and \((k^*, c^*)\).
Baseline parameters and \((k^*, c = 0, \hat{c})\).
Baseline parameters and \((k^*, c = 1, \hat{c})\).
IMPACT ON TRADING VOLUME

$k^*$

maturity $T$

Optimal collateral
IM and full collateral
IM only
OPTIMAL CONTINGENT COLLATERAL RULES

Same problem as before, same close-out convention, but now:

- larger set of admissible collateral rules $\mathcal{A}_c$

  $$C_t^A = \int_0^t c^A(s, W^A_s, -Z_s, C^A_s) dZ_s$$

- symmetry conditions $C_t^A = -C_t^B$
  $$c^A(t, W^A_t, -Z_t, C^A_t) = -c^B(t, W^B_t, Z_t, C^B_t) = -c^B(t, -W^A_t, Z_t, -C^A_t)$$

Results

- optimal collateral fraction $c^A_{t,*}$ independent of $(t, W^A_t)$, varies with $(Z_t, C^A_t)$
  - consistent with collateral triggers/thresholds observed in practice
  - suggests that CSA can take into account collateral performance (relevant for type/quality other than cash)

- same intuition as before, but larger utility gains
OPTIMAL CONTINGENT CSA

The diagram illustrates a 3D plot of the optimal contingent CSA, with axes labeled as Z and C. The values range from 0.8825 to 0.886, with a gradient color scale indicating different levels of probability or payoff. The plot shows a smooth transition with a slight peak, suggesting a moderate risk profile or optimal conditions for contingent CSA implementation.
THE COST OF (SUB)OPTIMAL COLLATERALIZATION

![Graph showing expected utility against maturity T for different collateral scenarios: optimal collateral fraction, IM and full collateral, IM only, no collateral, no default risk, and optimal CSA.](image)

- Expected Utility vs. Maturity T
- Maturity T values range from 1 to 10
- Expected Utility values range from -1 to -0.75
- Graph includes lines and markers for each scenario:
  - Optimal collateral fraction
  - IM and full collateral
  - IM only
  - No collateral
  - No default risk
  - Optimal CSA

The graph illustrates how the expected utility changes with maturity T for various collateral rules.
DISCUSSION

Funding costs

- agents can borrow at rate \( r + \lambda \)
  - agent is ITM: borrows at rate \( r \) instead of \( r + \lambda \) (earns a spread \( \lambda \))
  - agent is OTM: collateral funded at cost \( \lambda \) (only \( r \) is rebated); allocation to risky asset lower as borrowing is costly

- overall collateral lower due to opportunity cost of posting collateral

- fungibility of collateral crucial
DISCUSSION

**Funding costs**
- agents can borrow at rate $r + \lambda$
  - agent is ITM: borrows at rate $r$ instead of $r + \lambda$ (earns a spread $\lambda$)
  - agent is OTM: collateral funded at cost $\lambda$ (only $r$ is rebated); allocation to risky asset lower as borrowing is costly
- overall collateral lower due to opportunity cost of posting collateral
- fungibility of collateral crucial

**Rehypothecation vs. segregation** of collateral
- immaterial in the baseline model
  - costless borrowing, $r$ is rebated
- relevant when allowing for funding costs
  - larger utility losses / trading volume impact with segregation and suboptimal collateralization
OUTLINE

1. CSA design
2. A Model
3. Optimal Collateral Rules
4. Longevity swaps
5. Dynamic adverse selection
6. Conclusion
LONGEVITY SWAPS EXAMPLES

Stylized example: single payment at time $T > 0$

- notional $n > 0$, fixed payment $\bar{p} \in (0, 1)$
- variable payment $S_T$ (realized survival rate)

$$n \times \bar{p}$$

Party H
(hedger)

$$n \times S_T$$

Party HS
(hedge supplier)
LONGEVITY SWAPS EXAMPLES

Stylized example: single payment at time $T > 0$

- notional $n > 0$, fixed payment $\bar{p} \in (0, 1)$
- variable payment $S_T$ (realized survival rate)

Hedger’s viewpoint, cash collateral

- default spreads $(\lambda^H_t)_{t \geq 0}$, $(\lambda^{HS}_t)_{t \geq 0}$
- collateral fraction, $(c_t)_{t \geq 0}$, of market value, $(V_t)_{t \geq 0}$
- net (of rebates) collateral cost, $(\delta_t)_{t \geq 0}$
LONGEVITY SWAPS EXAMPLES

Stylized example: single payment at time $T > 0$

- notional $n > 0$, fixed payment $\bar{p} \in (0, 1)$
- variable payment $S_T$ (realized survival rate)

\[
V_0 = E^Q \left[ \exp \left( - \int_0^T (r_t + \Gamma_t)dt \right) (S_T - \bar{p}) \right]
\]

\[
\Gamma_t := \begin{cases} 
(1 - c^H_t)\lambda^H_t - \delta_t c^H_t & \text{if } V_t < 0 \\
(1 - c^{HS}_t)\lambda^{HS}_t - \delta_t c^{HS}_t & \text{if } V_t \geq 0 
\end{cases}
\]
Lee-Carter mortality improvements quantiles.
LONGEVITY SWAP MARGINS

Swap margins, $\frac{\overline{p}}{E^p[S_T]} - 1$. Hedger’s credit quality lower than hedge supplier’s ($\Delta = 100$ bps).
LONGEVITY SWAP MARGINS

Swap margins, $\frac{\overline{p}}{E_p[S_T]} - 1$. Hedger's credit quality lower than hedge supplier's ($\Delta = 100$ bps).
Swap margins, $\frac{\bar{p}}{E^p[S_T]} - 1$. Hedger and hedge supplier have the same credit quality.
ONE-WAY VS. TWO-WAY COLLATERALIZATION

Swap margins, $\frac{\bar{p}}{E^{\bar{p}}[S_T]} - 1$. Hedger’s credit quality lower than hedge supplier’s ($\Delta = 100$ bps).
Swap margins, \( \frac{\bar{p}}{E^p[S_T]} - 1 \). Hedger’s credit quality lower than hedge supplier’s (\( \Delta = 100 \) bps).
Swap margins, $\frac{\bar{p}}{E^p[S_T]} - 1$. Hedger's credit quality lower than hedge supplier's ($\Delta = 100$ bps).
DYNAMIC ADVERSE SELECTION

Adverse selection at inception
- different static types (good health vs. bad health)
- contract design may allow separation of types (Rotschild/Stiglitz)

Dynamic adverse selection
- homogenous policyholders at inception
- stochastic intensity of mortality $\mu^i_t$ (same law, not same process)
- different realizations of $\mu^i_t$ (and other relevant variables) make different guarantees/benefits relatively more or less attractive
- exogenous and endogenous surrender

Endogenous frailty representation, $\mu^\text{portf},i_t(u) = \mu^\text{pop}_t \eta^i_t(u)$
- $u \in \mathcal{U}$, contractual guarantees space $\mathcal{U}$
- $\eta^i_t(u)$ endogenous frailty (think of change of measure)
Average frailty for endowment assurance with different death benefits (endogenous surrender).
Average frailty for UL endowment (endogenous and exogenous surrender).
CONCLUSION

New interesting problems in contract design

- **CSA design** matter; rationale for CSAs observed in practice
- Collateral flows (variation margins) matter
- Collateralization is costly even in the absence of funding costs ("defaulter’s pay" form of credit enhancement)
- **Endogenous mortality** risk in life insurance contracts
- Impact of contract design: dynamic frailty representation

Other interesting questions

- Role of close-out conventions
- Equilibrium **CSA pricing**
- Gauging the gains from the reduction in aggregate risk
The talk is based on the following papers

- Bauer, D., E. Biffis, and L.R. Sotomayor (2013), Optimal collateralization with bilateral default risk, to be posted on SSRN


- Benedetti, D., and E. Biffis (2013), Insurance contract design and endogenous frailty, to be posted on SSRN
THANK YOU