Evaluation of proportional portfolio insurance strategies

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Origin of Portfolio Insurance

**Portfolio Insurance**

- **Leland and Rubinstein (1976), The Evolution of Portfolio Insurance**
  - Observation
    - "After the decline of 1973–74, many pension funds had withdrawn from the market (only to miss the rally in 1975)"
  - Idea
    - "If only insurance were available, those funds could be attracted back to the market"

- **Brennan and Schwartz (1976), The Pricing of Equity Linked Life Insurance Policies with an Asset Value Guarantee**

  - Repeated revival of portfolio insurance (PI)
  - Increasing commercial feasibility (decreasing costs of trading and product innovations)
**OBPI versus CPPI**

**Option based portfolio insurance (OBPI)**
- Protection **with options**
  - Protective put strategies (static or rolling)
  - Synthetic option strategies
  - **Kinked solution**

**Constant proportion portfolio insurance (CPPI)**
- Protection **without options**
  - Dynamic portfolio of underlying and risk-free asset
  - Cushion $C$ management technique
  - Cushion $= \text{difference between portfolio value } V \text{ and floor } F$
  - Leverage/multiplier $m$
  - Exposure $E$ in the risky asset:
    \[ E = m \times C \]
  - **Smooth solution**
Important results

→ Diffusion model setup (no jumps)
→ Objective: Maximize expected utility

**OBPI**

- El Karoui et al. (2005)
  → Terminal wealth constraint
  → Optimal solution: *Reduction of initial investment* (to finance the put option), apply optimal portfolio weights from the unrestricted problem to the reduced initial investment

**(C)PPI**

→ Terminal guarantee defines a subsistence level (*floor is growing with the risk–free interest rate*)
→ Optimal solution: use optimal portfolio weights from the unrestricted problem as multiple (apply them to the cushion)
Advantages (disadvantages) of (C)PPI method

- Trade off between risk and return
  - PI investor must give up upward participation to achieve the downward protection

- Disadvantage of (C)PPI
  - Asymptotically, the investor gives up more upward participation than OBPI investor
  - Put option is cheaper than zero bond (kinked vs smooth solution)

- Advantage of (C)PPI
  - Simple investment rule (less demanding than synthesizing an option payoff)
  - Easy to explain to the customer
  - (C)PPI can be applied to an infinite investment horizon
Recent developments or popular features in (C)PPI investments

- **Constraints on the investment level**
  - Minimum level of investment in the risky asset

- **Constraints on the leverage**
  - Borrowing restrictions

- **Variable and ’straight–line’ floors**
  - Locking in of profits (*ratcheting*)

- **Variable multiples**
  - Products allow for the multiple to vary over time in relation to the volatility of the risky asset
Outline of the further talk

- Optimality of (constant) proportion portfolio insurance strategies
  - Optimization criteria
  - Black and Scholes model (constant multiple)
  - Stochastic volatility models (constant vs variable multiple)
  - Evaluation of CPPI (constant multiple) vs PPI (time varying multiple) by means of real data
    - (Joint work with Sven Balder and Daniel Zieling)

- Transaction costs
  - Impacts of transaction costs (deterministic trading dates)
  - Optimal trading filter (stochastic trading dates)
  - Evaluation of trigger strategies w.r.t. performance measures (other than the optimization objective)
    - (Joint work with Sven Balder)

- Conclusion and further research
## Optimization criteria

**Examples: Main objectives**
- Expected utility
- Special case: Expected growth rate (*logarithmic utility*)
- Performance measures

**Examples: Additional constraints on**
- (Maximal and/or minimal) investment fraction
- (Maximal) shortfall probability (VaR, expected shortfall)
- (Maximal) turnover

→ **Keep it simple:** Consider the *growth rate of the cushion* (*logarithmic utility*) without additional restrictions

\[
\frac{1}{T} \ln \frac{C_T}{C_0}
\]
Growth rate (Black and Scholes model)

- Black and Scholes model (constant drift $\mu$ and volatility $\sigma$) for the index dynamics $S$
  
  \[ \frac{1}{T} \ln \frac{S_T}{S_0} \sim \mathcal{N}(\tilde{\mu}, \sigma) \text{ where } \tilde{\mu} = \mu - \frac{1}{2} \sigma^2 \]

- Consider a constant leverage $m$ (on the cushion)
  
  \[ \frac{1}{T} \ln \frac{C_T^m}{C_0^m} = \phi(m) + m \left( \frac{1}{T} \ln \frac{S_T}{S_0} \right) \]
  
  where $\phi(m) = -(m - 1) \left( r + \frac{1}{2} m \sigma^2 \right)$
Leverage $m$ implies a correction term:

\[
\begin{align*}
< 0 & \quad \text{for } m > 1 \quad \text{convex strategy} \\
= 0 & \quad \text{for } m = 1 \quad \text{linear strategy} \\
> 0 & \quad \text{for } m < 1 \quad \text{concave strategy}
\end{align*}
\]

Convex strategy (momentum strategy):

→ *Buy high and sell low*
→ Performance is penalized by round-turns of the risky asset
→ Is only optimal if the volatility is not too high (in comparison to the excess return of the risky asset)

Growth optimal leverage:

\[
m^* = \frac{1}{2} + \frac{\tilde{\mu} - r}{\sigma^2} = \frac{\mu - r}{\sigma^2}
\]
Illustration – Expected (cushion) growth rate

BS parameter: $\mu = 0.096$, $\sigma = 0.15$, $r = 0.03$

Optimal multiple $m^* = 2.93$
Stochastic volatility

Stochastic volatility (no jumps!)

- Diffusion setup for asset $S$ and variance dynamics $\sigma^2$
- Correction term ($\sigma$ stochastic)

$$\phi_{t,T}^{sv}(m) = - (m - 1) \left( r + \frac{1}{2} m \bar{\sigma}^2_{t,T} \right)$$

where $\bar{\sigma}_{t,T} = \sqrt{\frac{1}{T - t} \int_t^T \sigma^2_u \, du}$

- Optimal multiplier
  - No inter–temporal hedging demand for logarithmic utility
  - Is given by the portfolio weights of an investor with a very short investment horizon (myopic demand)

$$m_t^{*,sv} = \frac{\mu_t - r_t}{\sigma_t^2} = \frac{\lambda_t}{\sigma_t^2}$$
Equity risk premium

- Usual assumption
  - Risk premium is proportional to the variance, i.e. \( \lambda_t = \bar{\lambda} \sigma^2_t \)
  - Sharpe ratio is increasing in volatility
- (One) alternative assumption
  - Risk premium is proportional to the volatility, i.e. \( \lambda_t = \bar{\lambda} \sigma_t \)
  - Sharpe ratio is constant

Implications for variable multiple strategies

- Products which allow for the multiple to vary over time in relation to the volatility of the risky asset
  - Can not outperform the optimal constant multiple under the usual assumption
  - Can outperform the CPPI if e.g. the Sharpe ratio is constant
Return data (S&P500 – price index)

- Bloomberg data for the time period 1980–2010
  - Daily simple returns
  - Number of observation 7573

- Interest rate data
  - Discount yields of T-Bills (91 days to maturity)

- Summary statistics

<table>
<thead>
<tr>
<th>Average excess return ($\mu - r$)</th>
<th>Standard deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0404527</td>
<td>0.18119</td>
<td>-0.77758</td>
<td>24.9149</td>
</tr>
</tbody>
</table>

- We evaluate yearly growth rates of PPI strategies
- Overlapping years, monthly starting dates
Selection of proportional insurance strategies

- **Benchmark strategies**
  - Static PPI strategy *(buy and hold strategy)* $m = 1$
  - CPPI strategy with $m = 3$

- **Growth optimal strategies**
  - **Optimal constant multiple** strategy
    
    $$m^{*, \text{const}} = \frac{\mu - r}{\sigma^2} = \frac{0.0404527}{0.18119^2} = 1.23221$$

  - **Variable multiplier** strategy
    - based on historical volatility and
    - based on average of historical vol. and long term vol.

    $$m_{t \text{, hist}}^{\text{var}} = m_{\text{const}} \frac{\sigma}{\sigma_{\text{hist}}}$$
    $$m_{t \text{, mix}}^{\text{var}} = m_{\text{const}} \frac{\sigma}{\sigma_{\text{mix}}}$$

    where $\sigma_{\text{hist}}$ is calculated by a window of 21 days prior to the calculation of $m$ and $\sigma_{\text{mix}} = \frac{\sigma_{\text{hist}} + \sigma}{2}$
Descriptive results

![Box plots showing growth rates from January 1980 to January 2010](image)

Legend:
- m=1
- m=3
- $m^* = 1.23221$
- $m_{\text{var,hist}}$
- $m_{\text{var,mix}}$

- **Motivation and Problem**
  - Constant vs variable multiples
  - Transaction Costs
- **Optimization criteria**
  - Growth optimal leverage
  - Equity risk premium
- **Conclusion and further research**
  - Return data (S&P500 – price index)
Descriptive results

Growth rates Jan 1980 to Jan 1990

- m=1
- m=3
- $m^* = 1.23221$
- $m_{\text{var hist}}$
- $m_{\text{var mix}}$

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Descriptive results

Growth rates Jan 1990 to Jan 2000

$m=1$
$m=3$
$m^* = 1.23221$
$m_{t, hist}$
$m_{t, mix}$
Descriptive results
### Mean yearly growth rates (and standard deviations) of selected PPI strategies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 1$</td>
<td>0.0153</td>
<td>0.0221</td>
<td>0.1082</td>
<td>$-0.0763$</td>
</tr>
<tr>
<td></td>
<td>(0.1733)</td>
<td>(0.1684)</td>
<td>(0.0914)</td>
<td>(0.2045)</td>
</tr>
<tr>
<td>$m = 3$</td>
<td>$-0.0581$</td>
<td>$-0.0402$</td>
<td>0.2669</td>
<td>$-0.3786$</td>
</tr>
<tr>
<td></td>
<td>(0.5970)</td>
<td>(0.5657)</td>
<td>(0.2631)</td>
<td>(0.7384)</td>
</tr>
<tr>
<td>$m^*,\text{const}$</td>
<td>0.0141</td>
<td>0.0227</td>
<td>0.1306</td>
<td>$-0.1011$</td>
</tr>
<tr>
<td></td>
<td>(0.2166)</td>
<td>(0.2093)</td>
<td>(0.1120)</td>
<td>(0.257436)</td>
</tr>
<tr>
<td>$m_{t}^{\text{var,\text{hist}}}$</td>
<td>0.0244</td>
<td>0.0355</td>
<td>0.1563</td>
<td>$-0.0922$</td>
</tr>
<tr>
<td></td>
<td>(0.2533)</td>
<td>(0.2738)</td>
<td>(0.1924)</td>
<td>(0.2441)</td>
</tr>
<tr>
<td>$m_{t}^{\text{var,mix}}$</td>
<td>0.0196</td>
<td>0.0271</td>
<td>0.1304</td>
<td>$-0.0826$</td>
</tr>
<tr>
<td></td>
<td>(0.2075)</td>
<td>(0.2206)</td>
<td>(0.1225)</td>
<td>(0.2231)</td>
</tr>
</tbody>
</table>

Mean growth rate of variable multiple strategy (hist. vola) is larger than the one of the *optimal* constant multiple (*but no significant results*)
Transaction costs

... are important in the context of PI strategies

→ Reduction (increase) of the asset exposure in falling (rising) markets
→ Investor suffers from any round–turn of the asset price
→ Volatility has a negative impact on the return
→ Effect is particularly severe if there are in addition transaction costs, i.e. the effect is even leveraged by the transaction costs

- **Intuition (PPI):** Growth optimal multiple under transaction costs is lower than without transaction costs

- **Comparison to OBPI:**
  → Accounting of transaction costs implies higher option prices
  → Reduction of initial investment (to finance the put option) is higher
  → Lower leverage
Discrete–time PPI implementation

- Equidistant set of discrete trading dates

\[ T = \{ t_0 = 0 < t_1 < \cdots < t_{n-1} < t_n = T \} \]

- Discrete–time cushion dynamics \textbf{without transaction costs}

\[
C_{t_{k+1}}^{\text{Dis}} = e^{r(t_{k+1} - \min\{\bar{\tau}, t_{k+1}\})} \prod_{i=1}^{\min\{\bar{\tau}, k+1\}} \left( m \frac{S_{t_i}}{S_{t_{i-1}}} - (m - 1)e^{r \frac{T}{n}} \right)
\]

- Discrete–time cushion dynamics \textbf{with proportional transaction costs}

(transaction costs are financed by a cushion reduction, \( C_{t_{k+1}^+} \) denotes the floor after transaction costs)

\[
C_{t_{k+1}^+} = C_{t_{k+1}} - m\theta \max\{ C_{t_{k+1}^+}, 0 \} - C_{t_k} \left( \frac{S_{t_{k+1}}}{S_{t_k}} \right)
\]
Cushion dynamics with transaction costs

Three cases
- Increasing exposure due to rising markets
- Reduction of exposure due to decreasing markets
- Cash–lock – gap event due to extreme decrease in asset prices

Formally
- For $C_{t_k}^+ \leq 0$ it follows $C_{t_{k+1}}^+ = C_{t_{k+1}} = e^{r \frac{T}{n}} C_{t_k}^+$
- Otherwise

$$C_{t_{k+1}}^+ = \begin{cases} 
C_{t_k}^+ \left( \frac{1+\theta}{1+\theta m} m \frac{S_{t_{k+1}}}{S_{t_k}} - \frac{m-1}{1+\theta m} e^{r \frac{T}{n}} \right) & \text{for } e^{r \frac{T}{n}} \leq \frac{S_{t_{k+1}}}{S_{t_k}} \\
C_{t_k}^+ \left( \frac{1-\theta}{1-\theta m} m \frac{S_{t_{k+1}}}{S_{t_k}} - \frac{m-1}{1-\theta m} e^{r \frac{T}{n}} \right) & \text{for } \frac{m-1}{m(1-\theta)} \leq e^{r \frac{T}{n}} < \frac{S_{t_{k+1}}}{S_{t_k}} \\
C_{t_k}^+ \left( (1 - \theta) m \frac{S_{t_{k+1}}}{S_{t_k}} - (m - 1) e^{r \frac{T}{n}} \right) & \text{for } \frac{S_{t_{k+1}}}{S_{t_k}} < \frac{m-1}{m(1-\theta)} e^{r \frac{T}{n}} 
\end{cases}$$

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Remark – Volatility adjustments

Remark – Volatility adjustments ($\Delta t = \frac{T}{n}$ is small)

- **OBPI:**
  - Adjustment of option price to (proportional) transaction costs
  - Leland (1985) approach: Option volatility is adjusted to $\theta$
    \[
    \sigma_{\text{adjusted}}^2 = \sigma^2 \left( 1 + \sqrt{\frac{2}{\pi}} \frac{\theta}{\sigma \sqrt{\Delta t}} \right)
    \]

- **PPI:**
  - Similar reasoning implies an adjusted multiple $m^*,\text{adjusted}$, i.e.
    \[
    m^*,\text{adjusted} = \frac{\mu - r}{\sigma^2} - \sqrt{\frac{2}{\pi}} \frac{\theta}{\sigma \sqrt{\Delta t}}
    \]
High turnovers are normally controlled by a trading filter

Example:

→ Use sequence of stopping times (trading dates) $\tau_i$
→ Refer to discounted price movements $\hat{R}_{t,T} := e^{-r(T-t)} \frac{S_T}{S_t}$
→ Define trading filter by

$$\tau_{i+1} = \inf \left\{ t \geq \tau_i \middle| \left\{ \hat{R}_{\tau_i,t} \geq (1 + \kappa) \right\} \cup \left\{ \hat{R}_{\tau_i,t} \leq (1 - \kappa) \right\} \right\}$$

→ $\kappa$ can take into account gap risk
Optimal trigger level

Optimization problem

\[ \kappa^*(m) := \arg\max_{\kappa \leq \kappa_{\text{max}}} E_{\tau_k} \left[ \frac{1}{\tau_{k+1} - \tau_k} \ln \frac{C_{\tau_k+1} +}{C_{\tau_k+}} \right] \]

where \( \kappa_{\text{max}} := 1 - \frac{m - 1}{m(1 - \theta)} \)

→ **Condition** \( \kappa \leq \kappa_{\text{max}} \) prohibits gap risk
→ **Black Scholes model**: Quasi closed-form solution
→ **Optimal trigger** \( \kappa^*(m) \) can be computed (tractably)
→ **Overall optimal multiplier and trigger combination**
Illustration – Optimal trigger level

Parameter setup

→ Parameters of the Black and Scholes model are

\[ \mu = 0.096, \sigma = 0.15 \text{ and } r = 0.03 \]

→ Proportional transaction costs with \( \theta = 0.001 \)

Optimal trigger level \( \kappa^*(m) \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \kappa^*(m) )</th>
<th>( \kappa_{\text{max}}(m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>0.06</td>
<td>0.50</td>
</tr>
<tr>
<td>2.93</td>
<td>0.07</td>
<td>0.34</td>
</tr>
<tr>
<td>4.00</td>
<td>0.06</td>
<td>0.25</td>
</tr>
<tr>
<td>6.00</td>
<td>0.05</td>
<td>0.17</td>
</tr>
<tr>
<td>8.00</td>
<td>0.04</td>
<td>0.12</td>
</tr>
</tbody>
</table>
### Performance measures

Consider impact of trigger trading w.r.t. other performance measures, i.e.

<table>
<thead>
<tr>
<th>Performance measure</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td>$\frac{E[V_T - V_0 e^{rT}]}{\sqrt{\text{Var}[V_T]}}$</td>
</tr>
<tr>
<td><strong>Omega measure with level $K$</strong></td>
<td>$\frac{E[\max{V_T - K, 0}]}{E[\max{K - V_T, 0}]}$</td>
</tr>
<tr>
<td><strong>Sortino ratio with level $K$</strong></td>
<td>$\frac{E[V_T - K]}{\sqrt{E[(\max{K - V_T, 0})^2]}}$</td>
</tr>
<tr>
<td><strong>Upside potential ratio</strong></td>
<td>$\frac{E[\max{V_T - K, 0}]}{\sqrt{E[(\max{K - V_T, 0})^2]}}$</td>
</tr>
</tbody>
</table>

→ Continuous–time trading and no transaction costs
→ Closed–form solutions for Black and Scholes model
Remark – Performance measures without transaction costs

Illustration – Performance measures without transaction costs

→ BS parameter: $\mu = 0.096, \sigma = 0.15, r = 0.03$

→ Investment horizon $T = 1$ year, terminal guarantee $G = 80$

→ Continuous–time strategies, initial investment $V_0 = 100$, level $K = V_0 e^{rT}$
Illustration – Performance (daily rebalancing)

<table>
<thead>
<tr>
<th>$m$</th>
<th>Growth rate cushion</th>
<th>Mean $V_T$</th>
<th>Stdv $V_T$</th>
<th>Sharpe ratio</th>
<th>Sortino ratio</th>
<th>Upside potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>m=1</td>
<td>0.083</td>
<td>104.584</td>
<td>3.711</td>
<td>0.415</td>
<td>0.948</td>
<td>1.433</td>
</tr>
<tr>
<td></td>
<td>(0.988)</td>
<td>(1.000)</td>
<td>(0.999)</td>
<td>(0.985)</td>
<td>(0.978)</td>
<td>(0.987)</td>
</tr>
<tr>
<td>m=2</td>
<td>0.110</td>
<td>106.126</td>
<td>8.020</td>
<td>0.384</td>
<td>0.969</td>
<td>1.487</td>
</tr>
<tr>
<td></td>
<td>(0.950)</td>
<td>(0.999)</td>
<td>(0.994)</td>
<td>(0.956)</td>
<td>(0.938)</td>
<td>(0.962)</td>
</tr>
<tr>
<td>m=2.93</td>
<td>0.112</td>
<td>107.559</td>
<td>12.725</td>
<td>0.355</td>
<td>0.987</td>
<td>1.536</td>
</tr>
<tr>
<td></td>
<td>(0.891)</td>
<td>(0.996)</td>
<td>(0.986)</td>
<td>(0.935)</td>
<td>(0.900)</td>
<td>(0.938)</td>
</tr>
<tr>
<td>m=4</td>
<td>0.0857</td>
<td>109.175</td>
<td>19.127</td>
<td>0.320</td>
<td>1.004</td>
<td>1.588</td>
</tr>
<tr>
<td></td>
<td>(0.762)</td>
<td>(0.993)</td>
<td>(0.974)</td>
<td>(0.910)</td>
<td>(0.857)</td>
<td>(0.911)</td>
</tr>
<tr>
<td>m=6</td>
<td>-0.045</td>
<td>112.079</td>
<td>35.205</td>
<td>0.257</td>
<td>1.029</td>
<td>1.675</td>
</tr>
<tr>
<td></td>
<td>(0.982)</td>
<td>(0.939)</td>
<td>(0.865)</td>
<td>(0.777)</td>
<td>(0.858)</td>
<td></td>
</tr>
<tr>
<td>m=8</td>
<td>-0.283</td>
<td>114.697</td>
<td>59.176</td>
<td>0.197</td>
<td>1.040</td>
<td>1.745</td>
</tr>
<tr>
<td></td>
<td>(0.965)</td>
<td>(0.893)</td>
<td>(0.823)</td>
<td>(0.697)</td>
<td>(0.803)</td>
<td></td>
</tr>
</tbody>
</table>

In bracket: Percentage of no transaction cost value

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Evaluation of proportional portfolio insurance strategies
### Illustration – Performance (daily vs trigger rebalancing)

<table>
<thead>
<tr>
<th>$m$</th>
<th>Growth rate</th>
<th>Mean $V_T$</th>
<th>Stdv $V_T$</th>
<th>Sharpe ratio</th>
<th>Sortino ratio</th>
<th>Upside potential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cushion $E[V_T]$</td>
<td>$\sqrt{\text{Var}[V_T]}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily rebalancing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2.93</td>
<td>0.112</td>
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<td>(0.938)</td>
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<td>m=8</td>
<td>-0.283</td>
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<td>59.176</td>
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<td>1.745</td>
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<td></td>
<td>(0.965)</td>
<td>(0.893)</td>
<td>(0.823)</td>
<td>(0.697)</td>
<td>(0.803)</td>
<td></td>
</tr>
<tr>
<td>Trigger trading with $\kappa = 0.07$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2.93</td>
<td>0.121</td>
<td>107.801</td>
<td>12.685</td>
<td>0.375</td>
<td>1.045</td>
<td>1.585</td>
</tr>
<tr>
<td></td>
<td>(0.964)</td>
<td>(0.999)</td>
<td>(0.995)</td>
<td>(0.979)</td>
<td>(0.966)</td>
<td>(0.979)</td>
</tr>
<tr>
<td>m=8</td>
<td>-0.295</td>
<td>117.863</td>
<td>59.872</td>
<td>0.247</td>
<td>1.287</td>
<td>1.976</td>
</tr>
<tr>
<td></td>
<td>(0.992)</td>
<td>(0.977)</td>
<td>(0.964)</td>
<td>(0.931)</td>
<td>(0.956)</td>
<td></td>
</tr>
<tr>
<td>Trigger trading with $\kappa = 0.04$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2.93</td>
<td>0.120</td>
<td>107.780</td>
<td>12.772</td>
<td>0.371</td>
<td>1.046</td>
<td>1.589</td>
</tr>
<tr>
<td></td>
<td>(0.955)</td>
<td>(0.999)</td>
<td>(0.994)</td>
<td>(0.973)</td>
<td>(0.958)</td>
<td>(0.974)</td>
</tr>
<tr>
<td>m=8</td>
<td>-0.232</td>
<td>117.548</td>
<td>62.130</td>
<td>0.233</td>
<td>1.313</td>
<td>2.001</td>
</tr>
<tr>
<td></td>
<td>(0.989)</td>
<td>(0.966)</td>
<td>(0.948)</td>
<td>(0.900)</td>
<td>(0.936)</td>
<td></td>
</tr>
</tbody>
</table>
Conclusion and further research – PPI with variable multiplier

- Simple PPI: Floor is growing with risk–free interest rate
  → Optimization problem can be formulated w.r.t. the cushion
- Rule based multiple $m = \frac{\mu - r}{\sigma^2}$ has its merits
  → Expected (cushion) growth maximizing strategy
- Interesting question: Constant or variable multiple (risk premium proportional to $\sigma^2$ or to $\sigma$)
- Further research is needed to exploit the data adequately
  → Bootstrap (simulation) technique
  → Trade–off between larger set of observations and prevailing the data structure
Conclusion and further research – Transaction costs

- **Transaction costs**
  - Impact is similar for both PPI and OPBI
  - Adjustment of multiple (adjustment of all-in volatility for option pricing)

- **Trading filter**
  - Do not use the same filter for different multiples
  - Black and Scholes model: Growth optimal trading filter is **tractable to implement**
  - It seems to be **robust** w.r.t. other **performance measures** (Sharpe ratio, Sortino ratio, upside potential ratio)
  - Question: *How robust is the optimal BS-trading filter w.r.t. real data?*
Deviations from simple PPI’s

→ Many products rely on a **variable floor**

→ Example (**ratcheting**)

\[ F_t = \alpha M_t = \alpha \max\{ M_0 e^{\lambda t}, V_s e^{\lambda (t-s)}; s \leq t \} \]

→ \( M_0 \) denotes the all–time–high at \( t = 0 \)

→ PPI products use \( \lambda = 0 \) instead of (the tractable) \( \lambda = r \)

→ We also need to consider the **capped version** of all strategies, i.e.

\[ E_t = \min\{ mC_t, wV_t \} \]