Evaluation of proportional portfolio insurance strategies

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Origin of Portfolio Insurance

Portfolio Insurance

- Leland and Rubinstein (1976), *The Evolution of Portfolio Insurance*
  - → Obervation
    "After the decline of 1973–74, many pension funds had withdrawn from the market (only to miss the rally in 1975)"
  - → Idea
    "If only insurance were available, those funds could be attracted back to the market"
- Brennan and Schwartz (1976), The Pricing of Equity Linked Life Insurance Policies with an Asset Value Guarantee

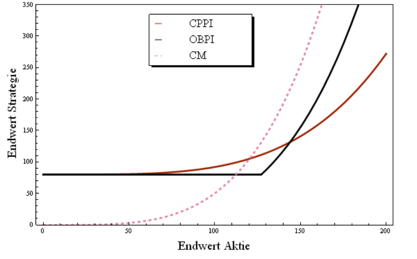
→ Repeated revival of portfolio insurance (PI)

→ Increasing commercial feasibility (decreasing costs of trading and product innovations)

OBPI versus CPPI

Option based portfolio insurance (OBPI)

- Protection **with options**
  - → Protective put strategies (static or rolling)
  - → Synthetic option strategies
  - → **Kinked solution**



Constant proportion portfolio insurance (CPPI)

- Protection without options
  - → Dynamic portfolio of underlying and risk–free asset
  - → Cushion C management technique
  - → Cushion = difference between portfolio value V and floor F
  - → Leverage/multiplier m
  - → Exposure E in the risky asset:

$$E = m \times C$$

  - → Smooth solution

Important results

→ Diffusion model setup (no jumps)

→ Objective: Maximize expected utility

OBPI

- El Karoui et al. (2005)
  - → Terminal wealth constraint
  - → Optimal solution: **Reduction of initial investment** (to finance the put option), apply optimal portfolio weights from the unrestricted problem to the reduced initial investment

(C)PPI

→ Terminal guarantee defines a subsistence level
(floor is growing with the risk–free interest rate)

→ Optimal solution: use optimal portfolio weights from the unrestricted problem as multiple (apply them to the **cushion**)

Advantages (disadvantages) of (C)PPI method

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- Trade off between risk and return
  - → PI investor must give up upward participation to achieve the downward protection
- Disadvantage of (C)PPI
  - → Asymptotically, the investor gives up more upward participation than OBPI investor
  - → Put option is cheaper than zero bond (**kinked vs smooth solution**)
- Advantage of (C)PPI
  - → Simple investment rule (less demanding than synthesizing an option payoff)
  - → Easy to explain to the customer
  - → (C)PPI can be applied to an infinite investment horizon

Recent developments

Recent developments or popular features in (C)PPI investments

- Constraints on the investment level
  - $\longrightarrow$ Minimum level of investment in the risky asset
- Constraints on the leverage
  - $\longrightarrow$ Borrowing restrictions
- Variable and 'straight–line' floors
  - $\longrightarrow$ Locking in of profits (*ratcheting*)
- Variable multiples
  - $\longrightarrow$ Products allow for the multiple to vary over time in relation to the volatility of the risky asset

Outline of the further talk

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- Optimality of (constant) proportion portfolio insurance strategies
  - → Optimization criteria
  - → Black and Scholes model (constant multiple)
  - → Stochastic volatility models (constant vs variable multiple)
  - → Evaluation of CPPI (**constant multiple**) **vs** PPI (**time varying multiple**) by means of real data
  - → (Joint work with Sven Balder and Daniel Zieling)
- Transaction costs
  - → Impacts of transaction costs (deterministic trading dates)
  - → **Optimal trading filter** (stochastic trading dates)
  - → Evaluation of trigger strategies w.r.t. performance measures (other than the optimization objective)
  - → (Joint work with Sven Balder)
- Conclusion and further research

Optimization criteria

Optimization criteria

- Examples: Main objectives
  - → Expected utility
  - → Special case: Expected growth rate (*logarithmic utility*)
  - → Performance measures
- Examples: Additional constraints on
  - → (Maximal and/or minimal) investment fraction
  - → (Maximal) shortfall probability (VaR, expected shortfall)
  - → (Maximal) turnover

→ **Keep it simple:** Consider the growth rate of the cushion (*logarithmic utility*) without additional restrictions

$$\frac{1}{T} \ln \frac{C_T}{C_0}$$

Growth rate (Black and Scholes model)

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- Black and Scholes model (constant drift μ and volatility σ) for the index dynamics S

  $\longrightarrow$ Growth rate of **buy and hold** strategy

$$\frac{1}{T}\ln\frac{S_T}{S_0} \sim \mathcal{N}(\tilde{\mu},\sigma) \text{ where } \tilde{\mu} = \mu - \frac{1}{2}\sigma^2$$

- Consider a constant **leverage** m (on the cushion)

  $\longrightarrow$ Cushion dynamics C is also lognormal
  $\longrightarrow$ Growth rate of **leveraged** strategy (cushion)

$$\frac{1}{T}\ln\frac{C_T^m}{C_0^m} = \phi(m) + m\left(\frac{1}{T}\ln\frac{S_T}{S_0}\right)$$

$$\text{where } \phi(m) = -(m-1)\left(r + \frac{1}{2}m\sigma^2\right)$$

Growth optimal leverage (Black and Scholes model)

Growth optimal leverage (Black and Scholes model)

- **Leverage** m implies a correction term

$$\begin{cases} < 0 & \text{for} \quad m > 1 \ \textbf{convex strategy} \\ = 0 & \text{for} \quad m = 1 \ \textbf{linear strategy} \\ > 0 & \text{for} \quad m < 1 \ \textbf{concave strategy} \end{cases}$$

- **Convex strategy** (momentum strategy)
  - → *Buy high* and *sell low*
  - → Performance is penalized by round–turns of the risky asset
  - → Is only optimal if the volatility is not too high (in comparison to the excess return of the risky asset)
- **Growth optimal leverage**

$$m^* = \frac{1}{2} + \frac{\tilde{\mu} - r}{\sigma^2} = \frac{\mu - r}{\sigma^2}$$

Illustration – Expected (cushion) growth rate

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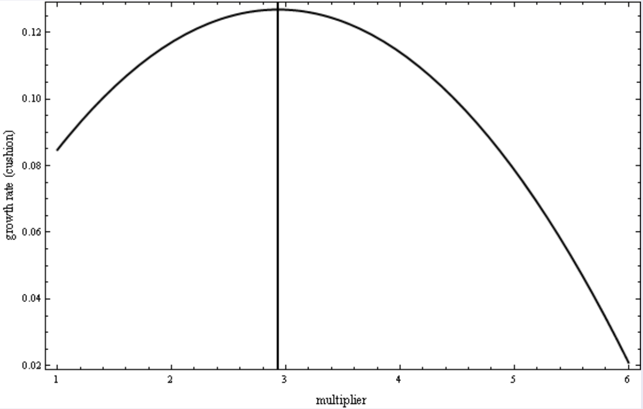
→ BS parameter: $\mu = 0.096$, $\sigma = 0.15$, $r = 0.03$

→ Optimal multiple $m^* = 2.93$



Stochastic volatility

Stochastic volatility (no jumps!)

- Diffusion setup for asset S and variance dynamics σ^2
- Correction term (σ stochastic)

$$\phi^{sv}_{t,T}(m) = -(m-1)\left(r + \frac{1}{2} m \bar{\sigma}^2_{t,T}\right)$$

$$\text{where } \bar{\sigma}_{t,T} = \sqrt{\frac{1}{T-t}\int_t^T \sigma_u^2 \, du}$$

- Optimal multiplier
  - $\longrightarrow$ No inter–temporal hedging demand for logarithmic utility
  - $\longrightarrow$ Is given by the portfolio weights of an investor with a very short investment horizon (myopic demand)

$$m_t^{*,sv} = \frac{\mu_t - r_t}{\sigma_t^2} = \frac{\lambda_t}{\sigma_t^2}$$

Equity risk premium

Equity risk premium

- Usual assumption
  - $\rightarrow$ Risk premium is proportional to the variance, i.e. $\lambda_t = \bar{\lambda}\sigma_t^2$
  - $\rightarrow$ Sharpe ratio is increasing in volatility
- (One) alternative assumption
  - $\rightarrow$ Risk premium is proportional to the volatility, i.e. $\lambda_t = \bar{\lambda}\sigma_t$
  - $\rightarrow$ Sharpe ratio is constant

Implications for variable multiple strategies

- Products which allow for the multiple to vary over time in relation to the volatility of the risky asset
  - $\rightarrow$ **Can not** outperform the optimal constant multiple under the usual assumption
  - $\rightarrow$ **Can** outperform the CPPI if e.g. the Sharpe ratio is constant

Return data (S&P500 – price index)

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- Bloomberg data for the time period 1980–2010
  - → Daily simple returns
  - → Number of observation 7573
- Interest rate data
  - → Discount yields of T-Bills (91 days to maturity)
- Summary statistics

Average excess return $(\mu - r)$	Standard deviation	Skewness	Kurtosis
0.0404527	0.18119	-0.77758	24.9149

→ We evaluate yearly growth rates of PPI strategies

→ Overlapping years, monthly starting dates

Selection of proportional insurance strategies

- Benchmark strategies
  - $\longrightarrow$ Static PPI strategy (*buy and hold strategy*) $m = 1$
  - $\longrightarrow$ CPPI strategy with $m = 3$
- **Growth optimal strategies**
  - $\longrightarrow$ **Optimal constant multiple** strategy

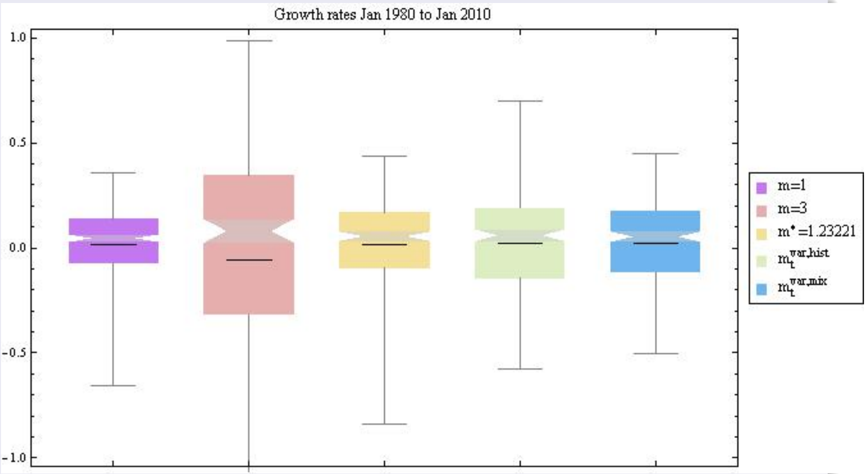
$$m^{*,\text{const}} = \frac{\mu - r}{\sigma^2} = \frac{0.0404527}{0.18119^2} = 1.23221$$

  - $\longrightarrow$ **Variable multiplier** strategy
    – based on historical volatility and
    – based on average of historical vol. and long term vol.

$$m_t^{\text{var, hist}} = m^{\text{const}} \frac{\sigma}{\sigma^{\text{hist}}}, \quad m_t^{\text{var, mix}} = m^{\text{const}} \frac{\sigma}{\sigma^{\text{mix}}}$$

where σ^{hist} is calculated by a window of 21 days prior to to the calculation of m and $\sigma^{\text{mix}} = \frac{\sigma^{\text{hist}} + \sigma}{2}$

Descriptive results



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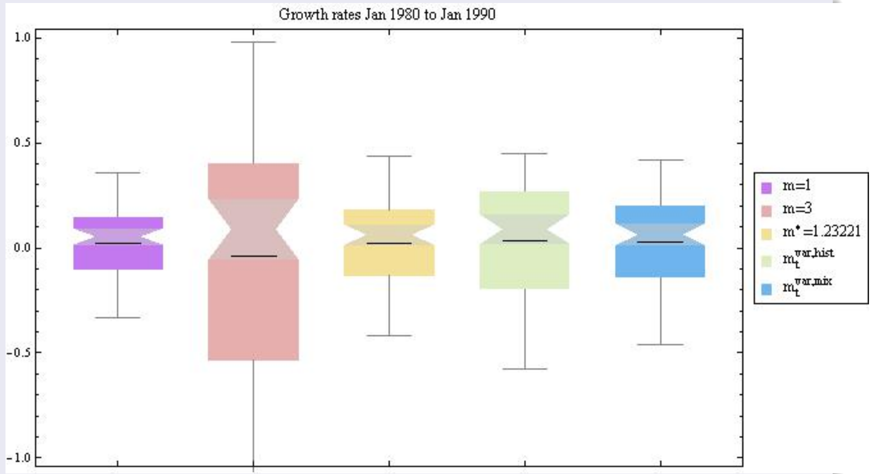

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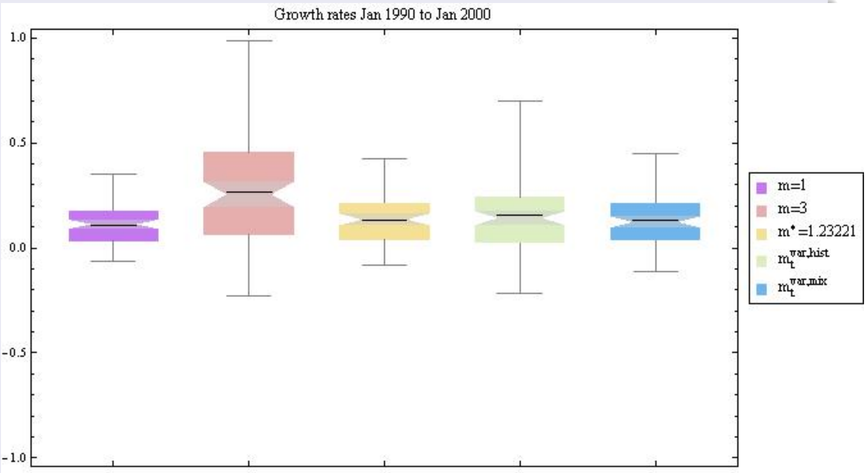

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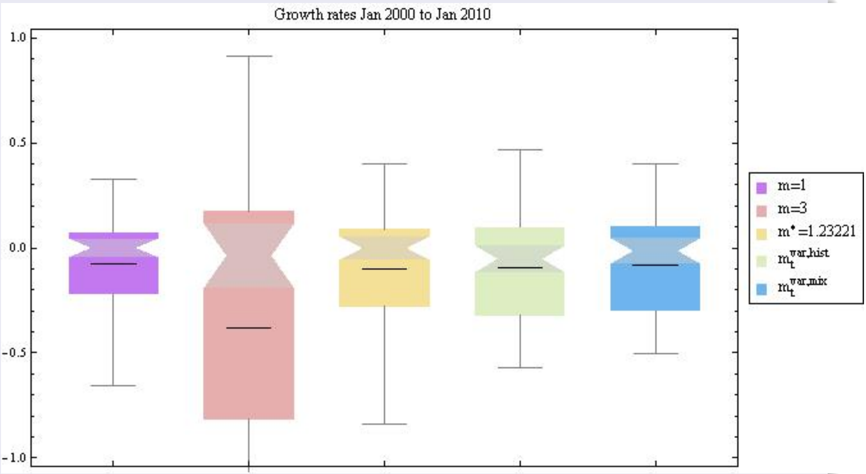

Mean yearly growth rates

Mean yearly growth rates (and standard deviations) of selected PPI strategies

	1980–2010	1980–1990	1990–2000	2000–2010
$m = 1$	0.0153 (0.1733)	0.0221 (0.1684)	0.1082 (0.0914)	**−0.0763** (0.2045)
$m = 3$	−0.0581 (0.5970)	−0.0402 (0.5657)	**0.2669** (0.2631)	−0.3786 (0.7384)
$m^{*,\text{const}}$	*0.0141* (0.2166)	*0.0227* (0.2093)	*0.1306* (0.1120)	*−0.1011* (0.257436)
$m_t^{\text{var,hist}}$	**0.0244** (0.2533)	**0.0355** (0.2738)	0.1563 (0.1924)	−0.0922 (0.2441)
$m_t^{\text{var,mix}}$	0.0196 (0.2075)	0.0271 (0.2206)	0.1304 (0.1225)	−0.0826 (0.2231)

→ Mean growth rate of variable multiple strategy (hist. vola) is larger than the one of the *optimal* constant multiple (**but no significant results**)

Transaction costs

Transaction costs

... are important in the context of PI strategies

→ Reduction (increase) of the asset exposure in falling (rising) markets
→ Investor suffers from any round–turn of the asset price
→ Volatility has a negative impact on the return
→ Effect is particularly severe if there are in addition transaction costs, i.e. the effect is even leveraged by the transaction costs

- **Intuition (PPI):** Growth optimal multiple under transaction costs is lower than without transaction costs
- **Comparison to OBPI:**

→ Accounting of transaction costs implies higher option prices
→ Reduction of initial investment (to finance the put option) is higher
→ Lower leverage

Discrete–time PPI implementation

Discrete–time PPI implementation

- Equidistant set of discrete trading dates

$$\underline{T} = \{t_0 = 0 < t_1 < \cdots < t_{n-1} < t_n = T\}$$

- Discrete–time cushion dynamics without transaction costs

$$C^{\text{Dis}}_{t_{k+1}} = e^{r(t_{k+1} - \min\{\tilde{\tau}, t_{k+1}\})} C^{\text{Dis}}_{t_0} \prod_{i=1}^{\min\{\tilde{\tau}, k+1\}} \left(m \frac{S_{t_i}}{S_{t_{i-1}}} - (m-1) e^{r\frac{T}{n}} \right)$$

- Discrete–time cushion dynamics with proportional transaction costs (transaction costs are financed by a cushion reduction, $C_{t_{k+1}+}$ denotes the floor after transaction costs)

$$C_{t_{k+1}+} = C_{t_{k+1}} - m\theta \left| \max\left\{ C_{t_{k+1}+}, 0 \right\} - C_{t_k+} \frac{S_{t_{k+1}}}{S_{t_k}} \right|$$

Cushion dynamics with transaction costs

Cushion dynamics with transaction costs

- Three cases
  - $\longrightarrow$ Increasing exposure due to rising markets
  - $\longrightarrow$ Reduction of exposure due to decreasing markets
  - $\longrightarrow$ Cash–lock – gap event due to extreme decrease in asset prices
- Formally
  - $\longrightarrow$ For $C_{t_k+} \leq 0$ it follows $C_{t_{k+1}+} = C_{t_{k+1}} = e^{r\frac{T}{n}} C_{t_k+}$
  - $\longrightarrow$ Otherwise

$$C_{t_{k+1}+} = \begin{cases} C_{t_k+}\left(\frac{1+\theta}{1+\theta m}\, m\frac{S_{t_{k+1}}}{S_{t_k}} - \frac{m-1}{1+\theta m}e^{r\frac{T}{n}}\right) & \text{for } e^{r\frac{T}{n}} \leq \frac{S_{t_{k+1}}}{S_{t_k}} \\ C_{t_k+}\left(\frac{1-\theta}{1-\theta m}\, m\frac{S_{t_{k+1}}}{S_{t_k}} - \frac{m-1}{1-\theta m}e^{r\frac{T}{n}}\right) & \text{for } \frac{m-1}{m(1-\theta)}e^{r\frac{T}{n}} \leq \frac{S_{t_{k+1}}}{S_{t_k}} < e^{r\frac{T}{n}} \\ C_{t_k+}\left((1-\theta)m\frac{S_{t_{k+1}}}{S_{t_k}} - (m-1)e^{r\frac{T}{n}}\right) & \text{for } \frac{S_{t_{k+1}}}{S_{t_k}} < \frac{m-1}{m(1-\theta)}e^{r\frac{T}{n}} \end{cases}$$

Remark – Volatility adjustments

Remark – Volatility adjustments ($\Delta t = \frac{T}{n}$ is small)

- **OBPI:**
  - $\longrightarrow$ Adjustment of option price to (proportional) transaction costs
  - $\longrightarrow$ Leland (1985) approach: **Option volatility** is **adjusted** to θ

$$\sigma^2_{\text{adjusted}} = \sigma^2 \left(1 + \sqrt{\frac{2}{\pi}} \frac{\theta}{\sigma\sqrt{\Delta t}}\right)$$

- **PPI:**
  - $\longrightarrow$ Similar reasoning implies an **adjusted multiple** $m^{*,\text{adjusted}}$, i.e.

$$m^{*,\text{adjusted}} = \frac{\mu - r}{\sigma^2} - \sqrt{\frac{2}{\pi}} \frac{\theta}{\sigma\sqrt{\Delta t}}$$

Trigger Trading

Trigger Trading

- High turnovers are normally controlled by a trading filter
- Example:
  - $\longrightarrow$ Use sequence of stopping times (trading dates) τ_i
  - $\longrightarrow$ Refer to discounted price movements $\hat{R}_{t,T} := e^{-r(T-t)}\frac{S_T}{S_t}$
  - $\longrightarrow$ Define trading filter by

$$\tau_{i+1} = \inf\left\{ t \geq \tau_i \,\middle|\, \left\{ \hat{R}_{\tau_i,t} \geq (1+\kappa) \right\} \cup \left\{ \hat{R}_{\tau_i,t} \leq (1-\kappa) \right\} \right\}$$

  - $\longrightarrow$ κ can take into account gap risk

Optimal trigger level

Optimal trigger level

- Optimization problem

$$\kappa^*(m) := \text{argmax}_{\kappa \le \kappa_{\max}} E_{\tau_k}\left[\frac{1}{\tau_{k+1}-\tau_k} \ln \frac{C_{\tau_{k+1}+}}{C_{\tau_k+}}\right]$$

where $\kappa_{\max} := 1 - \dfrac{m-1}{m(1-\theta)}$

$\longrightarrow$ Condition $\kappa \le \kappa_{\max}$ prohibits gap risk
$\longrightarrow$ Black Scholes model: Quasi closed–form solution
$\longrightarrow$ Optimal trigger $\kappa^*(m)$ can be computed (tractably)
$\longrightarrow$ Overall optimal multiplier and trigger combination

Illustration – Optimal trigger level

Parameter setup

→ Parameters of the Black and Scholes model are

$$\mu = 0.096,\ \sigma = 0.15 \text{ and } r = 0.03$$

→ Proportional transaction costs with $\theta = 0.001$

Optimal trigger level $\kappa^*(m)$

m	$\kappa^*(m)$	$\kappa_{\max}(m)$
$m = 2.00$	0.06	0.50
$m = 2.93$	0.07	0.34
$m = 4.00$	0.06	0.25
$m = 6.00$	0.05	0.17
$m = 8$	0.04	0.12

Performance measures

→ Consider impact of trigger trading w.r.t. other performance measures, i.e.

Performance measures

Sharpe ratio	$\frac{E\left[V_T - V_0 e^{rT}\right]}{\sqrt{\text{Var}[V_T]}}$
Omega measure with level K	$\frac{E[\max\{V_T - K, 0\}]}{E[\max\{K - V_T, 0\}]}$
Sortino ratio with level K	$\frac{E[V_T - K]}{\sqrt{E[(\max\{K - V_T, 0\})^2]}}$
Upside potential ratio	$\frac{E[\max\{V_T - K, 0\}]}{\sqrt{E[(\max\{K - V_T, 0\})^2]}}$

→ Continuous–time trading and no transaction costs

→ Closed–form solutions for Black and Scholes model

Remark – Performance measures without transaction costs

Illustration – Performance measures without transaction costs

- → BS parameter: $\mu = 0.096$, $\sigma = 0.15$, $r = 0.03$
- → Investment horizon $T = 1$ year, terminal guarantee $G = 80$
- → Continuous–time strategies, initial investment $V_0 = 100$, level $K = V_0 e^{rT}$

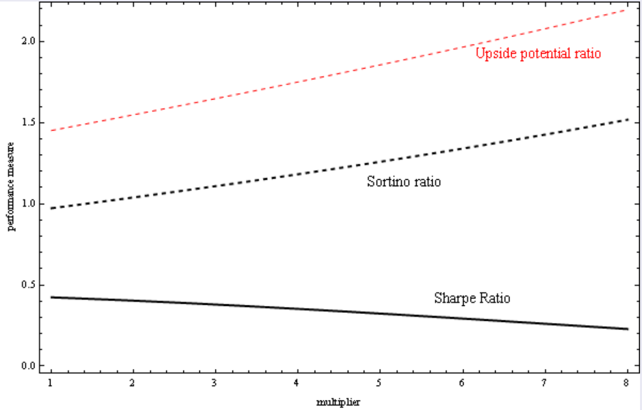

Illustration – Performance (daily rebalancing)

m	$G = 80$ and $\theta = 0.001$ **Daily rebalancing** Growth rate cushion	Mean V_T $E[V_T]$	Stdv V_T $\sqrt{\text{Var}[V_T]}$	Sharpe ratio	Sortino ratio	Upside potential
m=1	0.083	104.584	3.711	0.415	0.948	1.433
	(0.988)	(1.000)	(0.999)	(0.985)	(0.978)	(0.987)
m=2	0.110	106.126	8.020	0.384	0.969	1.487
	(0.950)	(0.999)	(0.994)	(0.956)	(0.938)	(0.962)
m=2.93	0.112	107.559	12.725	0.355	0.987	1.536
	(0.891)	(0.996)	(0.986)	(0.935)	(0.900)	(0.938)
m=4	0.0857	109.175	19.127	0.320	1.004	1.588
	(0.762)	(0.993)	(0.974)	(0.910)	(0.857)	(0.911)
m=6	-0.045	112.079	35.205	0.257	1.029	1.675
		(0.982)	(0.939)	(0.865)	(0.777)	(0.858)
m=8	-0.283	114.697	59.176	0.197	1.040	1.745
		(0.965)	(0.893)	(0.823)	(0.697)	(0.803)
	In bracket: Percentage of no transaction cost value					

Illustration – Performance (daily vs trigger rebalancing)

m	Growth rate cushion	Mean V_T $E[V_T]$	Stdv V_T $\sqrt{Var[V_T]}$	Sharpe ratio	Sortino ratio	Upside potential
	Daily rebalancing					
m=2.93	0.112	107.559	12.725	0.355	0.987	1.536
	(0.891)	(0.996)	(0.986)	(0.935)	(0.900)	(0.938)
m=8	-0.283	114.697	59.176	0.197	1.040	1.745
		(0.965)	(0.893)	(0.823)	(0.697)	(0.803)
	Trigger trading with $\kappa = 0.07$					
m=2.93	0.121	107.801	12.685	0.375	1.045	1.585
	(0.964)	(0.999)	(0.995)	(0.979)	(0.966)	(0.979)
m=8	-0.295	117.863	59.872	0.247	1.287	1.976
		(0.992)	(0.977)	(0.964)	(0.931)	(0.956)
	Trigger trading with $\kappa = 0.04$					
m=2.93	0.120	107.780	12.772	0.371	1.046	1.589
	(0.955)	(0.999)	(0.994)	(0.973)	(0.958)	(0.974)
m=8	-0.232	117.548	62.130	0.233	1.313	2.001
		(0.989)	(0.966)	(0.948)	(0.900)	(0.936)

Conclusion and further research – PPI with variable multiplier

Conclusion and further research – PPI with variable multiplier

- Simple PPI: Floor is growing with risk–free interest rate
  - $\longrightarrow$ Optimization problem can be formulated w.r.t. the cushion
- Rule based multiple $m = \frac{\mu - r}{\sigma^2}$ has its merits
  - $\longrightarrow$ Expected (cushion) growth maximizing strategy
- Interesting question: Constant or variable multiple (risk premium proportional to σ^2 or to σ)
- Further research is needed to exploit the data adequately
  - $\longrightarrow$ Bootstrap (simulation) technique
  - $\longrightarrow$ Trade–off between larger set of observations and prevailing the data structure

Conclusion and further research – Transaction costs

Conclusion and further research – Transaction costs

- Transaction costs
  - → Impact is similar for both PPI and OPBI
  - → Adjustment of multiple (adjustment of all–in volatility for option pricing)
- Trading filter
  - → Do not use the same filter for different multiples
  - → Black and Scholes model: Growth optimal trading filter is **tractable to implement**
  - → It seems to be **robust** w.r.t. other **performance measures** (Sharpe ratio, Sortino ratio, upside potential ratio)
  - → Question: *How robust is the optimal BS–trading filter w.r.t. real data?*

Conclusion and further research – Modifications of PPI

Conclusion and further research – Modifications of simple PPI

- Deviations from simple PPI's
  - $\longrightarrow$ Many products rely on a **variable floor**
  - $\longrightarrow$ Example (**ratcheting**)

$$F_t = \alpha M_t = \alpha \max\{M_0 e^{\lambda t}, V_s e^{\lambda(t-s)}; s \leq t\}$$

  - $\longrightarrow$ M_0 denotes the all–time–high at $t = 0$
  - $\longrightarrow$ PPI products use $\lambda = 0$ instead of (the tractable) $\lambda = r$
  - $\longrightarrow$ We also need to consider the *capped* **version** of all strategies, i.e.

$$E_t = \min\{mC_t, wV_t\}$$