Credit Risk: Risk Premia and Hedging

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Agenda

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Talk

This talk is based on the following papers:

- Bick, Hirsch, Kraft, Yildirim: Default and Idiosyncratic Risk Anomalies Revisited, Working Paper, Frankfurt and Syracuse.
- Bick, Kraft: Hedging Structured Credit Products during the Credit Crunch: A Horse Race of 10 Models, Working Paper, Frankfurt.

Topics

- This talk focuses on **measuring and hedging of credit risk**.
- First, we discuss the **estimation** of physical and risk-neutral default probabilities and the calculation of default premia.
- In our research, we studied whether default risk is correctly priced in the Fama-French model.
- This will only be a side-aspect in this talk.
- Second, we ask the question of whether **CDO tranches** could be **hedged** during the financial crisis.

Agenda

Types of Risks

A firm is exposed to several types of risks. The most important ones are

Market Risk: Risk related to movements in market variables such as stock indices, interest rates, exchange rates etc.

Credit Risk: Risk that a loss occurs because of a default by a counterparty.

Both are very **different** since

- default is a 0-1-event
- default risk is harder to measure
- default risk cannot be hedged away by a market index

How to Quantify Credit Risk?

There are **two** important **dimensions** of credit risk:

1. How likely is a default?
2. How big is the loss if a default occurs?

These dimensions are captured by the

1. **default probability** (PD),
2. **loss given default** (LGD).

Note that LGD = 1 - recovery rate.

Pricing of Defaultable Bonds

Payoff of a Defaultable Zero

$$p^d(T,T) = \begin{cases} 1 & \text{if} \quad \text{firm is solvent at } T \\ R = 1 - L & \text{if} \quad \text{firm is insolvent at } T \end{cases},$$

where R is the recovery rate and L the loss rate.

Key Insight

Prices are expected discounted payoffs under the **risk-neutral measure**.

Use this insight and calculate the price of a defaultable zero:

Price of a Defaultable Zero

$$p^d(0,T) = (\mathrm{SP}^Q \cdot 1 + \mathrm{PD}^Q \cdot R) \cdot e^{-rT} = (1 - L \cdot \mathrm{PD}^Q) \cdot p(0,T)$$

SP^Q and PD^Q are risk-neutral survival and default probabilities.

Risk-neutral Default Probabilities

You can use **bond data** to back out risk-neutral default probabilities:

$$\mathrm{PD}^Q = \frac{p(0,T) - p^d(0,T)}{L \cdot p(0,T)}$$

Example. Prices of zeros with maturity one year

$$p(0,1) = 0.97, \quad p^d(0,1) = 0.955$$

Rule of Thumb: Loss rate is 60%.

$$\Longrightarrow \mathrm{PD}^Q = \frac{p(t,T) - p^d(t,T)}{L \cdot p(t,T)} = \frac{0.97 - 0.955}{0.6 \cdot 0.97} = 0.0258,$$

i.e. the 1-year default probability of firm XY is 2.58% and thus the 1-year survival probability is 97.42%.

Risk-neutral vs. Physical Default Probabilities

In practice, we often need real (=physical) probabilities

$$\text{PD} = \text{Prob}(\text{"default"}),$$

instead of the risk-neutral probability

$$\text{PD}^Q = \text{Prob}^Q(\text{"default"}).$$

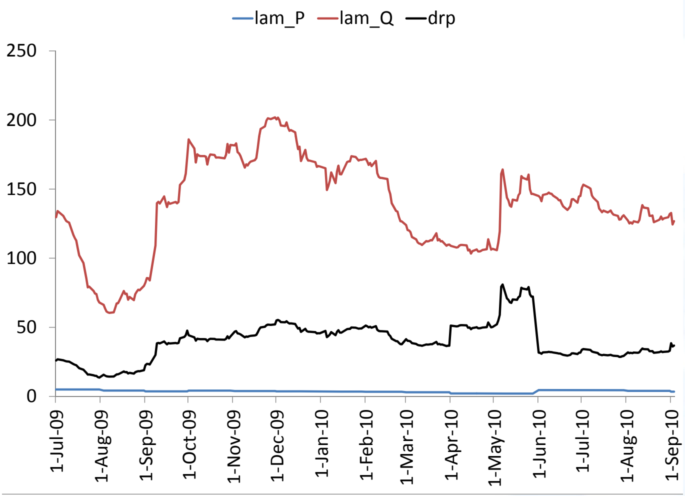
Rule

For pricing purposes, we use risk-neutral probabilities and, for risk management purposes, we use physical probabilities.

Crucial Q: How are these two probabilities related?

Empirical evidence shows that $\text{PD}^Q > \text{PD}$.

Risk-neutral vs. Physical Default Probabilities



Estimated actual and risk-neutral default intensities for Kraft Foods Inc.

Physical Default Probabilities

As we have seen, from bond prices we can back out risk-neutral default probabilities.

Q: How can we estimate the default probability of a firm?

Example "Unbalanced Coin": To estimate the probability of head, one can toss a coin several times (e.g. 100) and count the numbers of heads (e.g. 40).

$$\Longrightarrow \text{Prob}(\text{“head”}) = 40\%$$

Q: Can we apply this idea to estimate a firms's default probability?

Example "General Electric": For more than 100 years, we do not observe any default of GE.

$$\Longrightarrow \text{Prob}(\text{“default GE”}) = 0\% \qquad ???$$

This is obviously not true.

Physical Default Probabilities

Idea: Group firms into classes of firms with the same credit qualities and count the number of defaults in every class.

This leads to the concept of **ratings**.

Formally, a rating is nothing else than an **estimate of a firm's physical default probability** disguised in shortcuts like AA or B.

Estimating Physical Default Probabilities

- To estimate actual default probabilities, we apply a **dynamic logit approach**.
- We assume that the marginal probability of default over the next period is given by a logistic distribution

$$\mathrm{PD}(t-1,t) = \mathrm{Prob}_{t-1}(D_{it}=1) = \frac{1}{1+\exp(-\alpha-\beta x_{i,t-1})}$$

- D_{it} is a dummy variable equal to one if the firm defaults within month t and zero otherwise.
- $x_{i,t-1}$ represents a vector of explanatory variables.

Default information from Moodys's default risk service database (1986-2009): Rated Non-financial Firms

Year	Active Firms	Defaults	(%)
1986	868	9	1.03
1987	912	10	1.09
1988	856	10	1.16
1989	799	10	1.25
1990	745	19	2.54
1991	705	12	1.70
1992	747	7	0.93
1993	820	6	0.73
1994	906	4	0.44
1995	950	5	0.52
1996	1050	7	0.66
1997	1164	10	0.85
1998	1282	19	1.48
1999	1357	24	1.76
2000	1361	25	1.83
2001	1332	44	3.30
2002	1301	27	2.07
2003	1268	17	1.33
2004	1293	7	0.54
2005	1289	10	0.77
2006	1265	2	0.15
2007	1223	2	0.16
2008	1182	11	0.92
2009	1286	27	2.09

Variables Explaining Actual Defaults

Variable	Mean	Median	Std. Dev.	p25	p75
Panel A: Entire sample					
Rel. Net Income (NIMTAAVG)	.0032	.0056	.0133	.0009	.0101
Leverage (TLMTA)	.5676	.5654	.2401	.3818	.7775
Rel. Stock Return (EXRETAVG)	-.0029	.0001	.0384	-.0212	.0192
Stock Vol. (SIGMA)	.4186	.3399	.2670	.2393	.5028
Rel. Size (RSIZE)	-9.2308	-9.0282	1.5105	-10.1686	-7.8763
Rel. Cash (CASHMTA)	.0603	.0300	.07849	.0103	.0782
Market-to-book (MB)	2.0621	1.8068	1.4875	1.1084	2.2757
Trunc. Log Share (PRICE)	2.4743	2.7080	.5535	2.6119	2.7080
Panel B: Default subgroup					
Rel. Net Income (NIMTAAVG)	-.02856	-.02370	.0243	-.0482	-.0091
Leverage (TLMTA)	.8863	.9216	.0813	.8872	.9216
Rel. Stock Return (EXRETAVG)	-.0842	-.0871	.0567	-.1260	-.0427
Stock Vol. (SIGMA)	1.2003	1.326	.3232	.9765	1.4661
Rel. Size (RSIZE)	-12.3357	-12.5299	1.4814	-13.6115	-11.4249
Rel. Cash (CASHMTA)	.0570	.0311	.0701	.0127	.0728
Market-to-book (MB)	2.2531	1.0040	2.4860	.4483	2.2757
Trunc. Log Share (PRICE)	.2359	.2271	1.0631	-.4259	.9530

Default Prediction

Explanatory variable	Coefficient (z-value)
Rel. Net Income (NIMTAAVG)	-16.66***
	(-6.42)
Leverage (TLMTA)	5.89***
	(7.44)
Rel. Stock Return (EXRETAVG)	-4.51***
	(-4.02)
Stock Vol. (SIGMA)	1.97***
	(8.97)
Rel. Size (RSIZE)	-.099*
	(-1.95)
Rel. Cash (CASHMTA)	-4.51***
	(-5.41)
Market-to-book (MB)	.001
	(0.05)
Trunc. Log Share (PRICE)	-.431***
	(-5.80)
Constant	-13.28***
	(-15.41)
Observations	311,436
Defaults	324
Pseudo-R^2	0.3644

Calculating Default Premia

- Our estimated logit model can be used to generate a time series of physical default probabilities.
- From these default probabilities, one can calculate the **physical default intensity** λ^P

$$\mathrm{PD}(t, T) = 1 - e^{\lambda^P(T-t)} \approx \lambda^P(T-t).$$

- Fancier models for λ^P are possible!
- Eventually, we wish to calculate **default risk premia**:

$$\chi = \frac{\lambda^Q}{\lambda^P},$$

- Therefore, we need to estimate the **risk-neutral default intensity** λ^Q.
- Under the assumption of constant intensities, CDS quotes give an easy access to these intensities:

$$\mathrm{CDS} = \ell\lambda^Q,$$

- Again fancier models are possible!

Price Data: CDS Quotes from Markit

Year	Mean	Std. Dev.	p25	p50	p75	Observations
2001	141	224	48	83	149	45,576
2002	216	384	58	97	218	83,557
2003	153	282	36	61	146	97,601
2004	130	280	32	53	126	125,101
2005	136	384	29	48	117	133,155
2006	107	186	25	44	112	131,380
2007	121	177	28	50	140	149,740
2008	308	614	68	128	320	123,587
2009	418	1177	77	155	372	117,097
2010	209	479	66	112	206	89,288
Total	193	524	39	75	185	1,096,082

Annual Default Risk Premia

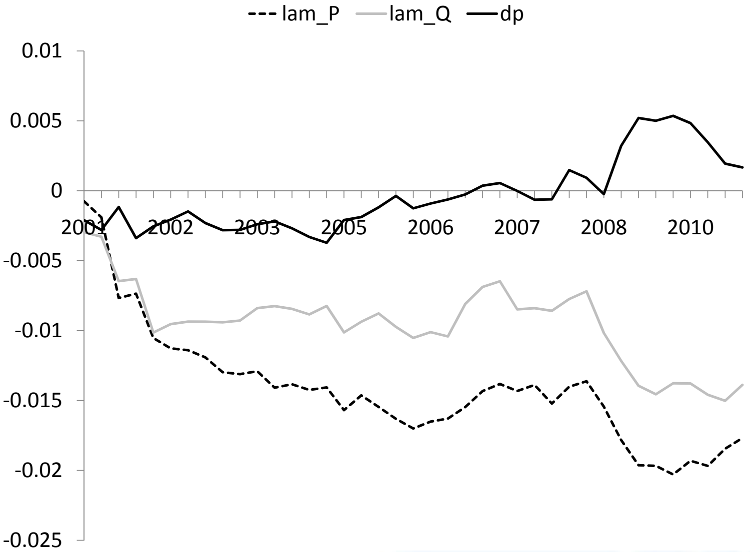
Year	Mean	Std. Dev.	p25	p50	p75	Observations
2001	64.04	107.45	10.01	24.81	66.20	24957
2002	99.80	183.89	11.35	31.65	94.92	48456
2003	78.84	158.49	10.80	27.47	77.36	56204
2004	93.14	183.43	15.88	42.15	103.18	72297
2005	107.20	220.79	18.75	50.23	122.88	76639
2006	107.27	227.14	18.00	47.03	109.28	75102
2007	127.11	263.51	21.22	55.41	134.45	86142
2008	140.92	362.50	18.59	58.94	144.87	73764
2009	78.46	155.33	10.92	33.39	85.15	86582
2010	120.92	154.23	29.71	71.80	148.39	73377
Total	105.24	222.02	15.85	45.05	113.87	673520

Fama-French Regressions: Portfolios Sorted on LAM P

	PF1	PF2	PF3	PF4	PF5	$PF5 - PF1$
mktrf	0.9863***	0.9552***	1.0026***	1.0195***	1.1004***	
	(77.9184)	(55.3096)	(43.5272)	(41.5917)	(47.3288)	
smb	-0.1627***	-0.0774***	-0.0582*	-0.1217***	0.1655***	
	(-8.8456)	(-3.9018)	(-1.7144)	(-4.3627)	(3.8725)	
hml	-0.3868***	0.0870	0.2958***	0.2263***	0.6994***	
	(-11.8191)	(1.5729)	(5.4369)	(3.6903)	(7.6093)	
umd	0.1263***	0.0426	0.1419***	-0.0773**	-0.3039***	
	(3.6763)	(1.0333)	(2.9310)	(-2.4275)	(-5.9120)	
alpha	0.0113	-0.03105	0.04025	-0.0220	-0.102**	-0.1133**
	(0.7918)	(-1.1371)	(1.6099)	(-0.6319)	(-2.1201)	(-2.1050)
size	16.69	16.10	15.85	15.53	14.61	
average return	0.0634	0.0581	0.1201	0.0871	0.0664	

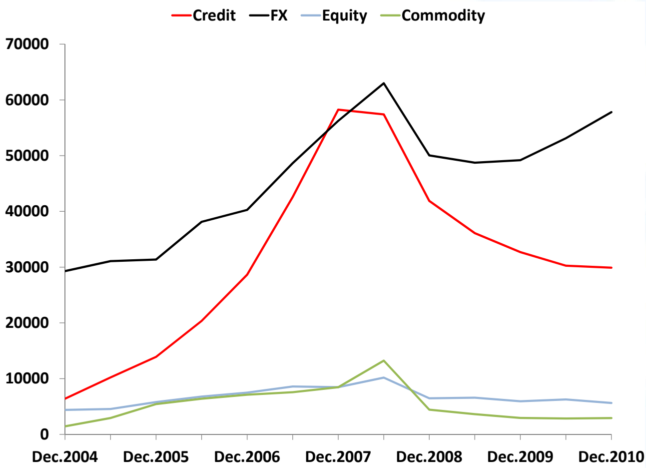
This is the so-called **default anomaly**.

Cumulated Alphas



Agenda

The Market for Credit Derivatives



Amounts outstanding of over-the-counter (OTC) derivatives, in billions of US dollars notional amounts outstanding
(Source: BIS Quarterly Review: September 2011)

What is a CDS Contract?

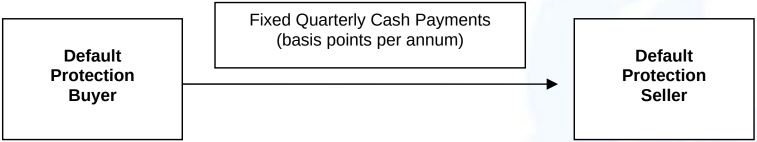
Credit Default Swap

The buyer of a CDS acquires protection from the seller against a default by a particular company or country (reference entity)

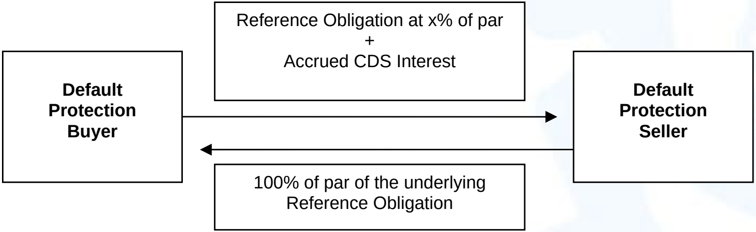
- A CDS contract can be interpreted as an **insurance contract against the default of a third party**.
- A credit default swap is an agreement between two counterparties that allows one counterparty to "long" a third-party credit risk, and the other counterparty to "short" the credit risk.
- The most controversial area of the specification of the contract is **definition of the credit event** (filing for bankruptcy, failure to pay, etc.)

CDS Mechanics

No Default



Default



Example: AT&T

- On 01/29/10 two counterparties A and B enter into a 5 year CDS on a notional value of USD 100m in which A pays B 82bps (=0.82%) per year, i.e. 820,000 dollars.
- The reference entity is AT&T Inc.
- If AT&T defaults at any time within that five years, then A will get an insurance payment from B.

Credit Indices: CDX and iTraxx

- The **CDX-NAIG** index is an equally weighted portfolio of 125 investment grade North American companies.
- The **CDX-NAHY** index is an equally weighted portfolio of 100 below investment grade North American companies.
- The **iTraxx Europe** is an equally weighted portfolio of 125 investment grade European companies.
- If the five-year CDX index is bid 96 offer 97, then a portfolio of 125 CDS contracts on the CDX companies can be bought for 97bps, e.g., USD 800,000 of 5-year protection on each name (total USD 100m) could be purchased for USD 970,000 per year. When a company defaults the annual payment is reduced by $1/125$.
- On 01/29/10 the spread on the CDX-NAHY was 575bps.

CDX-NAIG: Some Current Members

Markit CDX Indices

Constituents for CDX.NA.IG

Summary **Constituents**

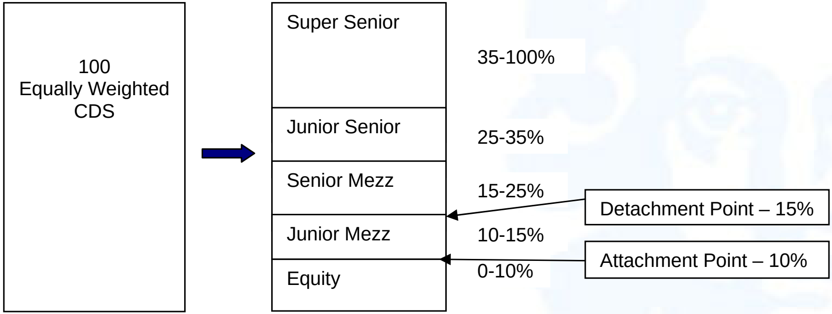
Index Constituents

Short Name	Entity CLIP	Reference Obligation	Av Rating	Sector	Weight
ACE Ltd	0A4848	ACE-INAHldgs 8.875 15Aug29	A	Financial	0.800%
Aetna Inc.	0A8985	AET 6.625 15Jun36 BondCall	A	Financial	0.800%
Alcoa Inc.	014B98	AA 5.72 23Feb19	BBB	Materials	0.800%
Altria Gp Inc	0C4291	MO 9.7 10Nov18	BBB	Consumer Stable	0.800%
Amern Elec Pwr Co Inc	027A8A	AEP 5.25 01Jun15	BBB	Utilities	0.800%
Amern Express Co	027D97	AXP 4.875 15Jul13	A	Financial	0.800%
Amern Intl Gp Inc	028EFB	AIG 6.25 01May36 Struc	BBB	Financial	0.800%
Amgen Inc.	0D4278	AMGN 4.85 18Nov14 BondCall	A	Consumer Stable	0.800%
Anadarko Pete Corp	0A3576	APC 5.95 15Sep16 BondCall	BBB	Energy	0.800%
Arrow Electrs Inc	0E69A8	ARW 6.875 01Jun18	BBB	Industrial	0.800%
AT&T Inc	0A226X	ATTINC 5.1 15Sep14	A	Communications and Technology	0.800%
AT&T Mobility LLC	0A232K	ATTINC-ML (4) 7.125 15Dec31	A	Communications and Technology	0.800%
Autozone Inc	0F8665	AZO 5.5 15Nov15 Struc	BBB	Consumer Cyclical	0.800%
Avnet, Inc.	058B87	AVT 6 01Sep15 BondCall	BBB	Industrial	0.800%
Barrick Gold Corp	06DG91	ABX 5.8 15Nov34	BBB	Materials	0.800%
Baxter Intl Inc	0H8994	BAX 6.625 15Feb28	A	Consumer Stable	0.800%
Boeing Cap Corp	09G715	BA-CapCorp 5.8 15Jan13	A	Financial	0.800%

CDO

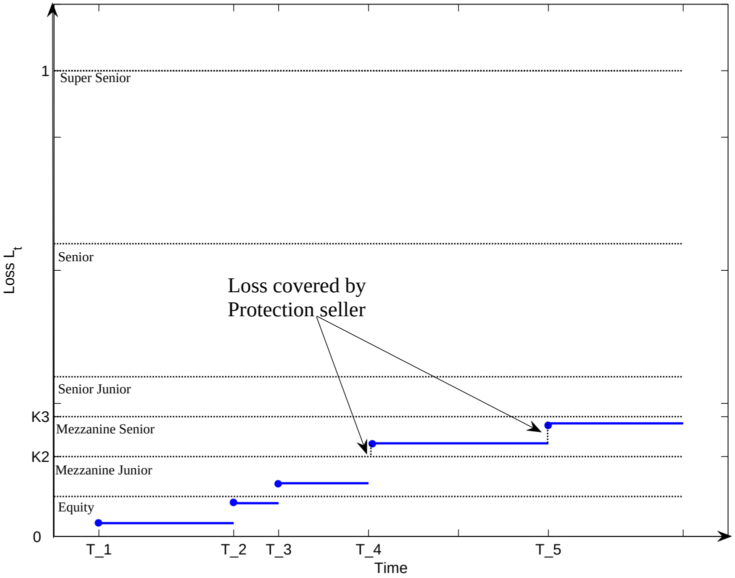
- The **underlying** of a CDO is a pool of loans, CDS contracts etc.
- The pool is sliced into **tranches**.
- The **cash flows** generated by the pool are used to service the tranches according to the seniority (waterfall).
- Therefore, the holder of a tranche receives interest payments, but has to cover losses that are attributed to the tranche.
- Like Credit Defaut Swaps, a CDO tranche can thus be split up into **two legs**:
  - **Fee leg**: Interest rate payments
  - **Protection leg**: Payments upon default

CDO Structuring



Source: Markit

CDO: Protection Legs



CDO: Loss Process

- At the **default time** τ_i of firm i, there is a **loss** of $\ell_i = (1 - R_i)N_i$ (expressed in dollars).
- The **total loss** until time t is given by

$$L_t \equiv \sum_{i=1}^{I} \ell_i \mathbf{1}_{\{\tau_i \leq t\}}$$

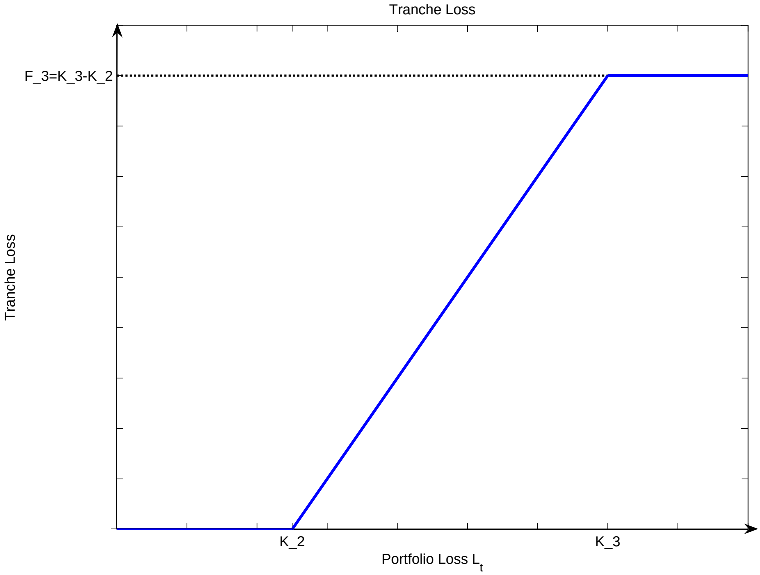
- We are also interested in the **tranche loss** of tranche m at time t.

Relative Tranche Loss

The relative loss of the tranche $[K_{m-1}, K_m]$ can be written as

$$L_t^m \equiv \frac{\max\{L_t - K_{m-1}; 0\} - \max\{L_t - K_m; 0\}}{K_m - K_{m-1}}.$$

Absolute Tranche Loss: Mezzanine



CDO Legs

Value of Fee Leg

$$V_t^{f,m} = \delta \sum_{t_k > t} p(t, t_k)(1 - \mathrm{E}_t[L_{t_k}^m]).$$

Value of Protection Leg

$$V_t^{p,m} = \sum_{t_k > t} p\left(t, \frac{t_{k-1}+t_k}{2}\right)\left(\mathrm{E}_t[L_{t_k}^m] - \mathrm{E}_t[L_{t_{k-1}}^m]\right).$$

- The fair spread is given by $s_t^m = \frac{V_t^{p,m}}{V_t^{f,m}}$
- **Challenges.** Calculate the expected percentage tranche losses $\mathrm{E}_t[L_t^m]$.

Agenda

Hedging CDO Tranches

- Practitioners like to hedge CDO tranches by CDS contracts (single name or index).
- Given a model one can (numerically) **calculate deltas** of tranches with respect to CDS contracts.
- This idea is similar to Black-Scholes delta hedging.
- We perform a **horse race of 10 models**.

Question

Did delta hedging work during the crisis?

10 Models

- **Goal:** Compute Default Probabilities
- **Bottom-up models**
  - Default times are modeled using an intensity model.
  - Aggregation of single-name default probabilities via model for correlation structure (e.g. Copula)
- **Top-down models**
  - Loss process L is modeled directly.
  - Contagion effects can be addressed in a convenient way.

Top-down Models

- **Longstaff-Rajan model**

$$L_t = 1 - e^{-\gamma_1 N_{1t}} e^{-\gamma_2 N_{2t}} e^{-\gamma_3 N_{3t}}$$

- **Self-exciting model**: Intensity of loss process

$$d\lambda_t = \kappa(\theta - \lambda_t)\, dt + \delta\, dL_t$$

- **Dynamical Generalized Poisson Loss Model**: Default process is given by (M_i Poisson)

$$N_t = \max\{Z_t, I\} \quad \text{with} \quad Z_t = \sum_{i=1}^{I} \alpha_i M_{it}$$

Calibration

- **To Do:** Calibrate all models to market data (CDO, CDS index, CDS)
- Bottom-up: Each tranche is represented by **one** correlation (implied correlation, base correlation)
- Top-down: Calibrate **several** parameters

$$\left(\sum_m \left(\frac{CDO_t^m - Mod_t^m(\Theta)}{CDO_t^m}\right)^2 + \left(\frac{Index_t - Mod_t^{ind}(\Theta)}{Index_t}\right)^2\right)^{1/2}$$

Calibration

Model	[0 – 3%]	[3 – 6%]	[6 – 9%]	[9 – 12%]	[12 – 22%]
Gau	0	0	0	0.01	0.26
Clay	0	0	0	0	0
DT (4/4)	0	0	0	0.01	2.49
DT (6/4)	0	0	0	0.01	2.03
DT (6/6)	0	0	0	0.01	1.85
T (4)	0	0	0	0	0.72
T (8)	0	0	0	0.01	0.85
T (12)	0	0	0	0.01	0.73

Average calibration error in percent implied by base correlations. The time period is 09/01/08 to 09/22/08.

Model	[0 – 3%]	[3 – 6%]	[6 – 9%]	[9 – 12%]	[12 – 22%]
SE	0	16.16	2.83	0.94	13.06
DGPL	0.01	0.85	1.47	6.46	3.49
LR	0.06	0.08	0.06	0.12	0.07

Average calibration error in percent across all top-down models. The time period is 09/01/08 to 09/22/08.

Hedging of CDO Tranches

- Calculate **deltas** $\psi^{m,index/CDS}$
- **P&L analysis**

$$PL_t = \Delta MTM_t^m - \psi_{t-1}^{m,index/CDS} \Delta MTM_t^{CDS/Index}$$

where MTM mark-to-market value.

Hedging Error

$$\frac{\text{Average absolute P\&L of the hedged position}}{\text{Average absolute P\&L of the unhedged position}}$$

Vola Residual

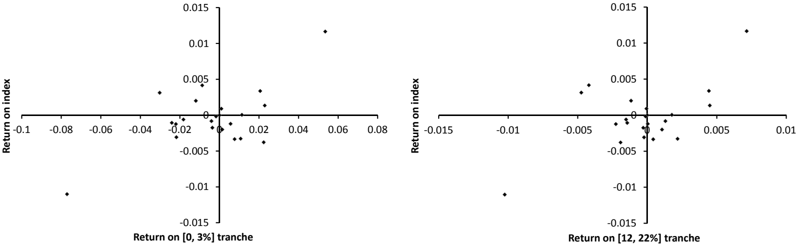
$$\frac{\text{Volatiltiy of the P\&L of the hedged position}}{\text{Volatiltiy of the P\&L of the unhedged position}}$$

Hedging of Tranches with CDS Contracts

Model	[0 − 3%]	[3 − 6%]	[6 − 9%]	[9 − 12%]	[12 − 22%]
	Index				
Gau	83.7 (75.27)	82.01 (76.35)	84.08 (77.28)	108.65 (95.31)	113.48 (118.23)
T(4)	82.5 (76.66)	83.39 (77.76)	79.64 (74.83)	96.83 (86.67)	94.02 (85.07)
SE	230.32 (223.69)	140.09 (131.43)	108.26 (102.53)	126.61 (117.22)	118.8 (108.53)
DGPL	165.74 (153.33)	91.66 (86.22)	84.5 (82.84)	104.42 (96.45)	106.5 (96.09)
	Portfolio of three single-name CDS				
Gau	75.08 (70.61)	69.89 (68.63)	71.92 (66.31)	114.87 (112.78)	110.4 (101.96)
T(4)	77.81 (74.37)	72.18 (69.55)	70.04 (68.99)	90.96 (78.94)	91.09 (89.14)
SE	194.11 (198.56)	111.04 (105.29)	83.65 (81.07)	96.61 (93.29)	94.19 (89.13)
DGPL	142.47 (138.93)	75.63 (72.53)	72.63 (70.32)	88.51 (81.71)	89.75 (83.74)

Hedging tranches with index CDS or a portfolio of three single-name CDS contracts in September 2008.

Empirical Dependence of Tranches and CDS Index



Daily index returns versus daily tranche returns.

Hedging of Tranches with Tranches

T(4)

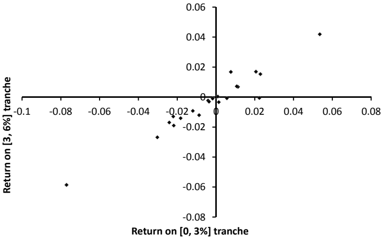
Hedging instrument	[0 − 3%]	[3 − 6%]	[6 − 9%]	[9 − 12%]	[12 − 22%]
[0 − 3%]	0 (0)	26.35 (27.21)	29.92 (33.42)	60.3 (61.34)	52.99 (52.99)
[3 − 6%]	26.3 (25.74)	0 (0)	22.27 (20.46)	48.85 (45.68)	46.18 (43.26)
[6 − 9%]	29.86 (33.86)	21.91 (20.88)	0 (0)	49.69 (52.03)	45.49 (41.74)
[9 − 12%]	98.46 (104.55)	80.89 (81.94)	87.18 (93.41)	0 (0)	46.65 (46.74)
[12 − 22%]	82.23 (85.82)	67.11 (67.04)	71.54 (71.37)	42.06 (40.13)	0 (0)

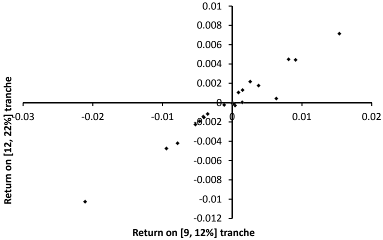
DGPL

Hedging instrument	[0 − 3%]	[3 − 6%]	[6 − 9%]	[9 − 12%]	[12 − 22%]
[0 − 3%]	0 (0)	57.31 (53.01)	59.2 (56.64)	59.3 (56.42)	61.07 (57.93)
[3 − 6%]	111.47 (105.3)	0 (0)	18.75 (16.6)	36.71 (31.04)	48.03 (41.64)
[6 − 9%]	116.59 (113.83)	19.24 (17.13)	0 (0)	29.51 (29.29)	40.95 (37.59)
[9 − 12%]	106.17 (100.43)	34.23 (28.32)	26.86 (25.52)	0 (0)	20.92 (21.03)
[12 − 22%]	108.31 (109.24)	42.41 (37)	34.94 (31.88)	19.66 (20.52)	0 (0)

Hedging tranches with tranches in September 2008.

Empirical Dependence of Tranches





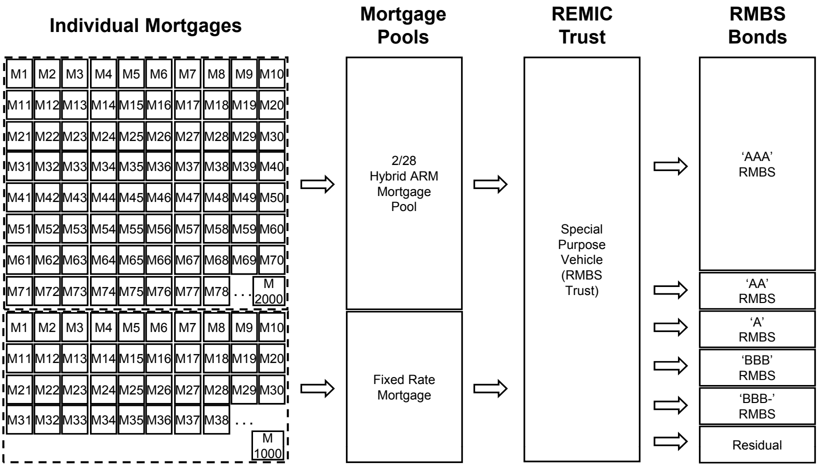
Daily tranche returns versus daily tranche returns.

Agenda

Conclusion

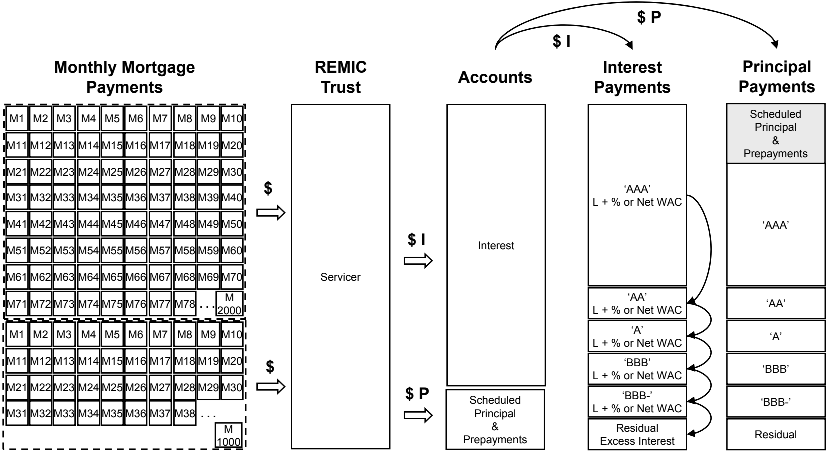
- We have discussed the estimation of physical and risk-neutral default probabilities.
- Prices can be used to back out implied risk-neutral default probabilities.
- A database with actual defaults is needed to estimate physical default probabilities.
- Hedging of CDO tranches with CDS contracts seems to be involved.
- During the crisis the relation between both was loose.
- Hedging of tranches with tranches works better, but such a hedge is hard to implement.
- Liquidity issues were not considered and make it even harder to set up hedges.

CDO Structuring: Mortgage-Backed Securities



Source: Fitch, RMBS: Residential Mortgage-Backed Securities, REMIC: Real Estate Mortgage Investment Conduit

CDO: Waterfall



Source: Fitch

Modeling Default Counter and Default Stopping Times

- We consider a portfolio consisting of I entities.
- The **default stopping times** of the entities are denoted by

$$\tau_1, \tau_2, \ldots, \tau_I$$

- The **number of defaults** is counted by the default process (**bottom up**)

$$N_t \equiv \mathbf{1}_{\{\tau_1 \le t\}} + \mathbf{1}_{\{\tau_2 \le t\}} + \cdots + \mathbf{1}_{\{\tau_I \le t\}}$$

- The ***k*-th jump time** of N is denoted by T_k.
- Therefore, we can also write (**top down**)

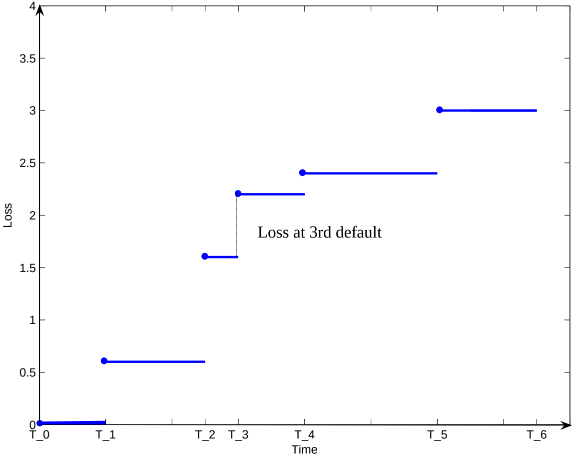
$$N_t = \sum_{k \ge 1} \mathbf{1}_{\{T_k \le t\}}$$

- The corresponding **loss process** reads

$$L_t = \sum_{k \ge 1} \mathbf{1}_{\{T_k \le t\}} \ell_k,$$

where ℓ_k is the loss associated with the k-th loss.

Loss Process



Losses are assumed to be 0.2, 0.6 or 1.0