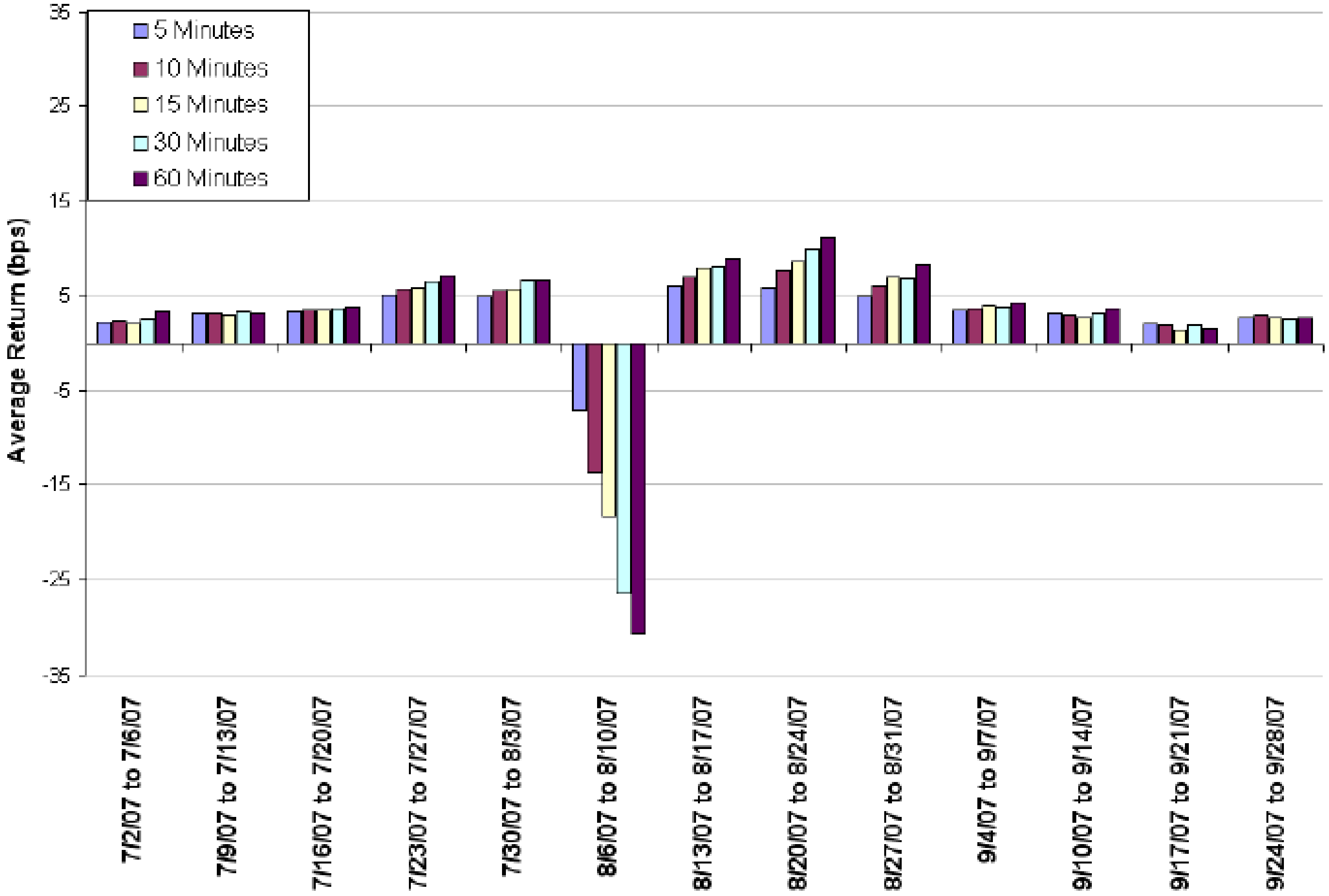
Different views on financial risk

- Journalistic view: tries to attribute each market move to an external (political, economic, corporate) event.
  Very useful for 'predicting' events.. after they occur.
- Stochastic models: represents market moves as random, and tries to match its features (volatility, correlation, distribution) using an exogenously specified statistical model.

Large market moves in absence of external shocks may be problematic to explain in such models: Oct 1987 crash, August 2007 hedge fund losses, May 2010 Flash Crash, unexpected peaks in volatility and correlations,...

In both of these views, large market moves which cannot be traced back to a significant external event are not 'explainable': they are dismissed as 'black swans' or 'outliers'.

Average return of a long-short market-neutral equity fund (July-Sept 2007). Source: Lo & Khandani (2009)



‘Black swans’: a convenient un-truth?

- Large unexpected jump in prices, volatilities and correlations frequently occur, even in liquid markets.
- Calling such events ’black swans’ (= unpredictable outliers) is convenient ex-post, since it implies that nothing can be done about them and removes any responsability/ blame from risk managers who fail to hedge against such events.
- However, the failure of statistical (or event-based) models to explain/predict the risk associated with such large market moves may simply be due to... lack of sophistication of the models.
- Even though stochastic models may be sophisticated in the statistical sense (many factors with complex dynamics..) they may simply omit... the underlying economic factors.

Back to Economics 101

- We claim that most of these events have in fact more basic, and less mysterious, explanations in terms of dynamics of market supply and demand.
- Most of these peaks and jumps in volatility, price and correlation may in fact be traced back to 'liquidity events': deleveraging of large portfolios, strategies followed by large asset managers, unwinding of large positions, hedging by option sellers,... which lead to systematic shifts in supply/demand and generate **endogenous shifts** in price dynamics, not attributable to any large external shock.
- We propose a framework for modeling such feedback phenomena and show that, in many relevant cases, the endogenous risk generated by these phenomena has a significant **predictable** component.

- In a sense, there is nothing new about this approach: it simply amounts to going back to modeling of price moves in term (good old) supply-demand dynamics.
- However, we will show through examples that, combined with the quantitative tools of stochastic modeling, such models of *endogenous risk* models provide useful tools for risk management which complement existing statistical approaches.

References

R Cont, L Pichard (2009) When diversification increases risk, Working Paper.

R Cont, L Wagalath (2010) Running for the exit, Working Paper.

R Cont, A Kukanov and S Stoikov (2010) The Price Impact of Order Book Events, `http://ssrn.com/abstract=1712822`

Correlation, correlation, correlation

Correlation if a crucial ingredient for quantifying the risk of portfolios and a key input for asset allocation.

Many investment strategies are based on a knowledge of the level of correlation.

Correlations and covariances between returns of assets, indices and funds are routinely estimated from historical data and used by market participants as inputs for trading, portfolio optimization and risk management.

BUT:

Market correlation is certainly one of the parameters we understand the **least**.

Whereas sophisticated models have been proposed for the distributional features of univariate returns but correlations are typically assumed to be constant and determined either from empirical covariances or indirectly by estimating factor models for fund returns.

These estimators implicitly assume that the underlying return correlations are **constants** which supposedly reflect structural dependencies between “fundamentals”, therefore stable in time.

Sample covariance and realized correlation

The most common method for estimating correlation between risk factors (indices, FX rates, interest rates,..) is to use historical covariance.

Prices $S_t^i, i = 1..n$ observed on a time grid
$t \in \{t_k = k\Delta, k = 0..\ T/\Delta\}$ (where for example $\Delta = 1$ day).

Given observations of (daily) returns $r_i(t)$ for risk factors, covariance between risk factors i and j is computed as

$$\hat{\sigma}_{ij} = \frac{\Delta}{T} \sum_{t=1}^{T} \overline{r_i}(t)\overline{r_j}(t) \qquad \hat{\sigma}_i = \sqrt{\hat{\sigma}_{ii}} \tag{1}$$

The sample correlation matrix is defined as:

$$\hat{\rho}_{ij} = \frac{\hat{\sigma}_{ij}}{\hat{\sigma}_i \hat{\sigma}_j} \in [-1, 1] \tag{2}$$

This *realized correlation* may vary from sample to sample.

Such “realized correlations” are routinely used for:

- Computation of risk indicators of portfolios: Value at Risk, risk measures
- Portfolio optimization, asset allocation and design of statistical arbitrage strategies
- Pricing and hedging of multi-asset derivatives: index options, options on baskets, spread options, multiasset equity options

in which they are identified with the “true” correlations, assumed to be constant.

Empirical facts about realized correlations

1. Equity returns and index are *positively correlated* in a significant way, across sectors and countries.
2. The precise level of these (realized) correlations strongly fluctuates in time.
3. Correlations across asset classes tend to be significantly stronger during periods of market illiquidity/crisis.
4. These observations are stable across countries and time periods.

These "stylized facts" need to be modeled and call for economic explanations but have no theoretical counterpart in most model which assume constant correlations.

Recently some attempts have been made to introduce randomness in correlation structures:

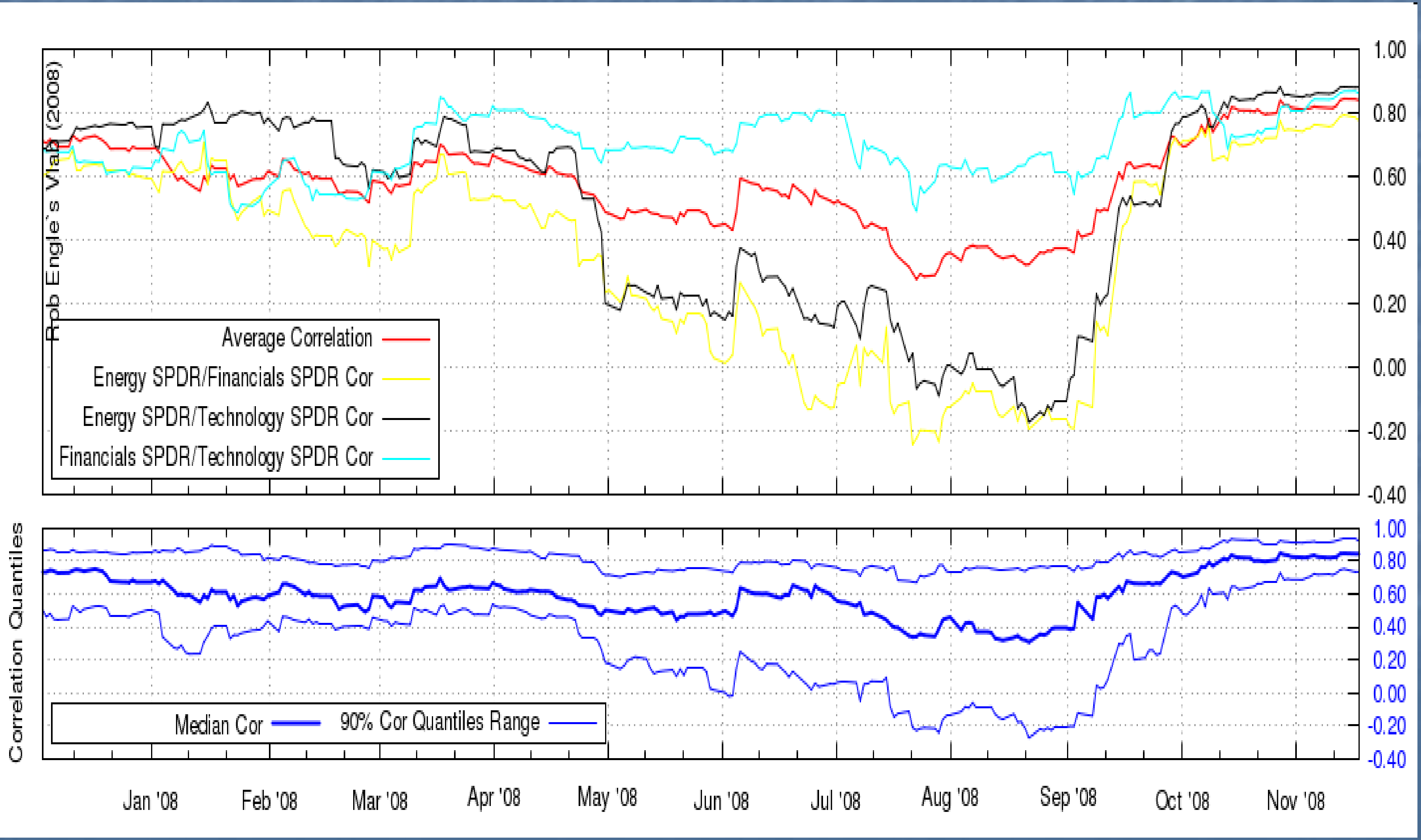
Factor models with time varying betas

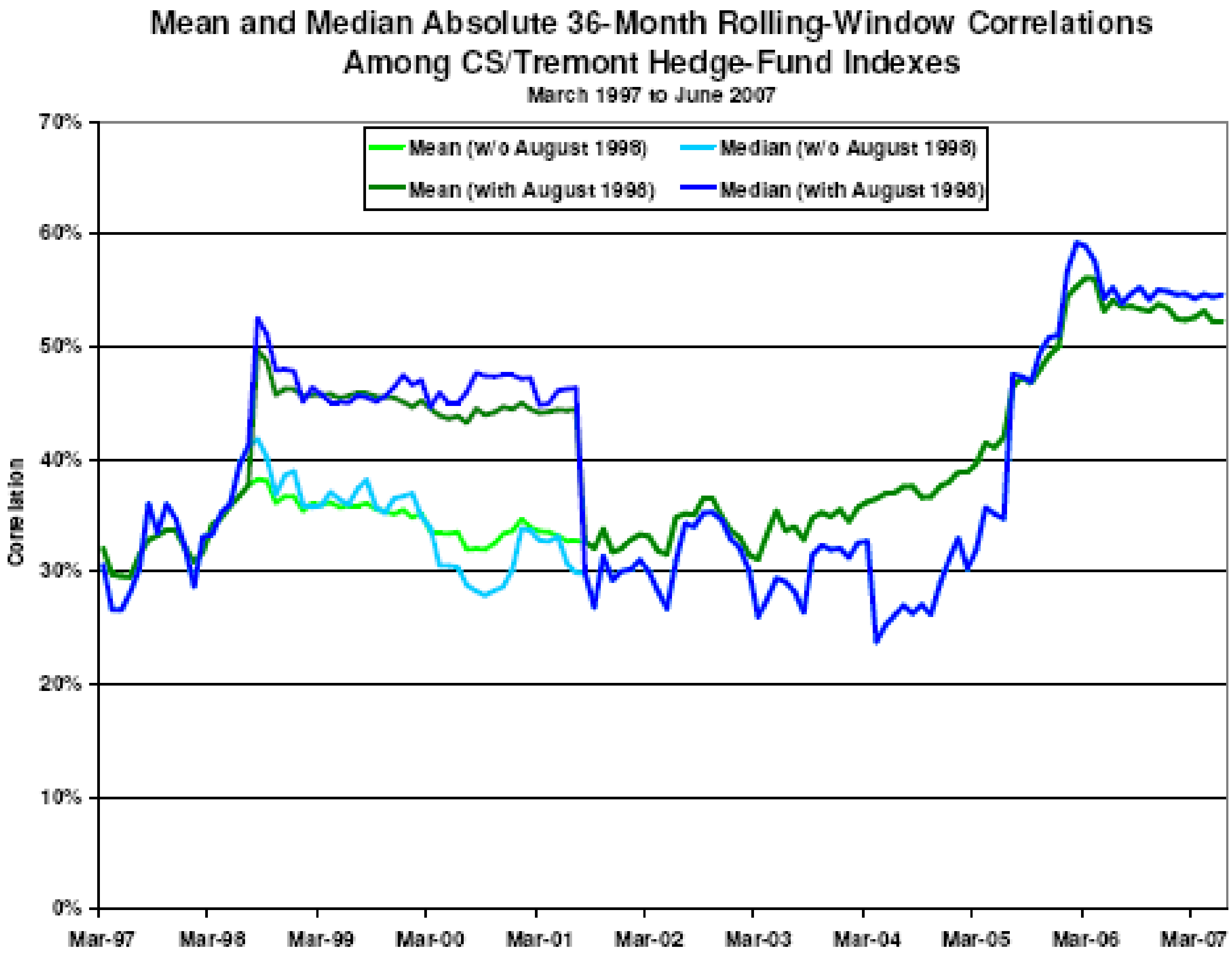
Regime switching models

Stochastic correlation models: DCC GARCH (Engle et al 2007, Engle 2009), Wishart processes (Gourieroux Jasiak 07)

BUT: no underlying economic interpretation, no link with liquidity

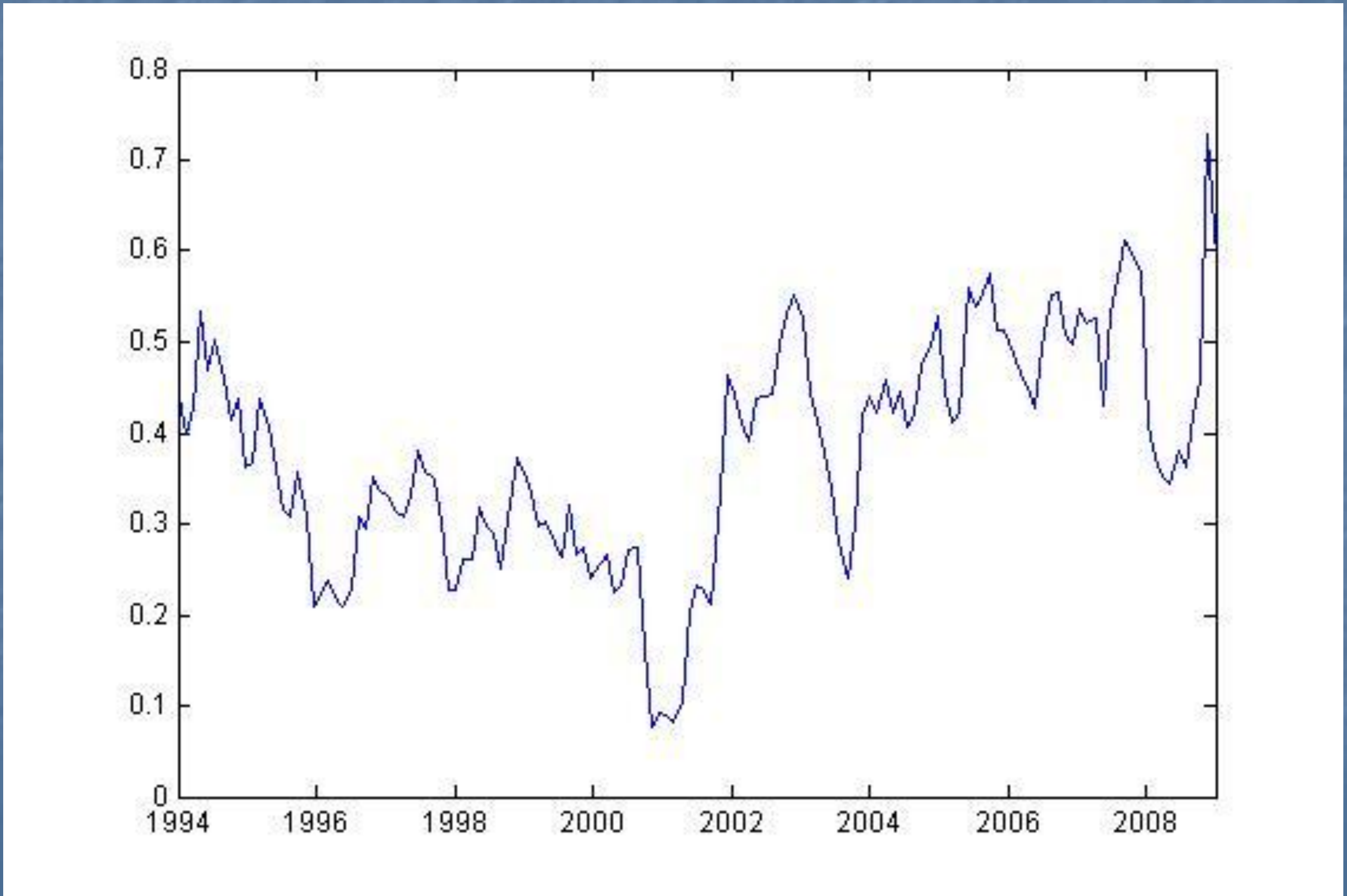
Realized correlations among US market sectors: 2007-2008





Mean and Median Absolute 36-Month Rolling-Window Correlations
Among CS/Tremont Hedge-Fund Indexes
March 1997 to June 2007
Mean (w/o August 1998)
Median (w/o August 1998)
Mean (with August 1998)
Median (with August 1998)
Correlation
70%
60%
50%
40%
30%
20%
10%
0%
Mar-97
Mar-98
Mar-99
Mar-00
Mar-01
Mar-02
Mar-03
Mar-04
Mar-05
Mar-06
Mar-07

Moving average realized correlations among CSFB Tremont Hedge Fund strategy indices



The economic origin of correlations in returns

Two different origins:

Correlation in fundamentals: common factors in returns (usual explanation)→ correlation in "fundamentals", should not vary strongly in time

Correlation from trading: generated by systematic supply/demand generated by specific (often rule-based) trading strategies → depends on market liquidity

Price impact from trading

Portfolio allocation of a fund $\phi^a(t) = (\phi_1^a(t), ..., \phi_n^a(t))$

Variation in portfolio from trading: $\Delta\phi^a(t) = \phi^a(t) - \phi^a(t-h)$
Aggregate excess demand of institutional investors/ fund managers:

$$\Delta\phi(t) = (\Delta\phi_i(t), i = 1..n) \quad = \quad \sum_{a\in\mathbb{A}} \Delta\phi^a(t) \tag{3}$$

We model the evolution of the price as the sum of a random component reflecting "fundamental" volatility + a price impact term due to institutional trading:

$$\frac{\Delta S_t^i}{S_t^i} = \underbrace{\mu_i\ h}_{\text{Expected return}} + \overbrace{f\Big(\frac{\Delta\phi_i(t)}{\underbrace{\lambda_i}_{\text{market depth}}}\Big)}^{\text{price impact function}} + \underbrace{\sigma_i\ \epsilon_i(t)}_{\text{random component}} \tag{4}$$

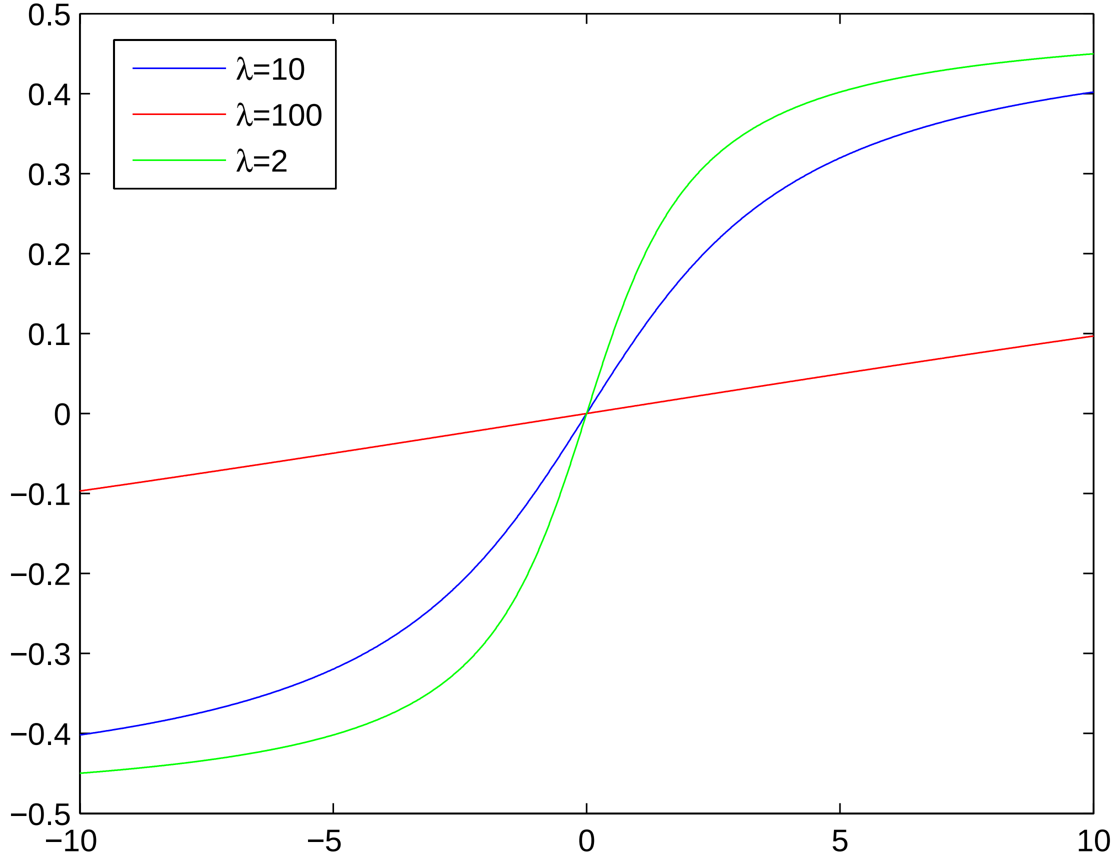


Figure 1: Price impact function

Example 1: fixed-mix strategies

A large proportion of mutual funds follow fixed-mix strategies.

Fund manager has a (medium term) target allocation in terms of proportions allocated to each asset *class* i.

To maintain the target allocation, the fund manager needs to rebalance periodically his/her portfolio as prices moves.

For a long-only portfolio this leads to buy if price decreases, sell if price increases, until strategic allocation is revised.

A model of price impact from fixed-mix strategies

Market moves due to fundamental factors (with correlation C^0: fundamental correlation).

Fund managers adjust portfolio to meet allocation target $X = (x_i)$

$$\phi_i^a(t_{k+1}) = \frac{x_i^a}{S_i^*(t_k)} \sum_j \phi_j^a(t_k+1) S_j^*(t_k) = \frac{x_i^a}{S_i^*(t_k)} \sum_j \phi_j^a(t_k) S_j^*(t_k) \quad (5)$$

Price impact of trades:

$$\frac{S_i(t_{k+1}) - S_i^*(t_k)}{S_i^*(t_k)} = \frac{\phi_i(t_{k+1}) - \phi_i(t_k)}{\underbrace{\lambda_i}_{\text{market depth}}} \quad (6)$$

Questions: what is the realized correlation among assets generated by these supply/demand patterns?

How does the realized correlation compare with the fundamental correlation?

Simulation example

3 asset classes

100 funds

Input noise covariance ("fundamental" covariance)

$$\Sigma = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.3 \end{pmatrix}$$

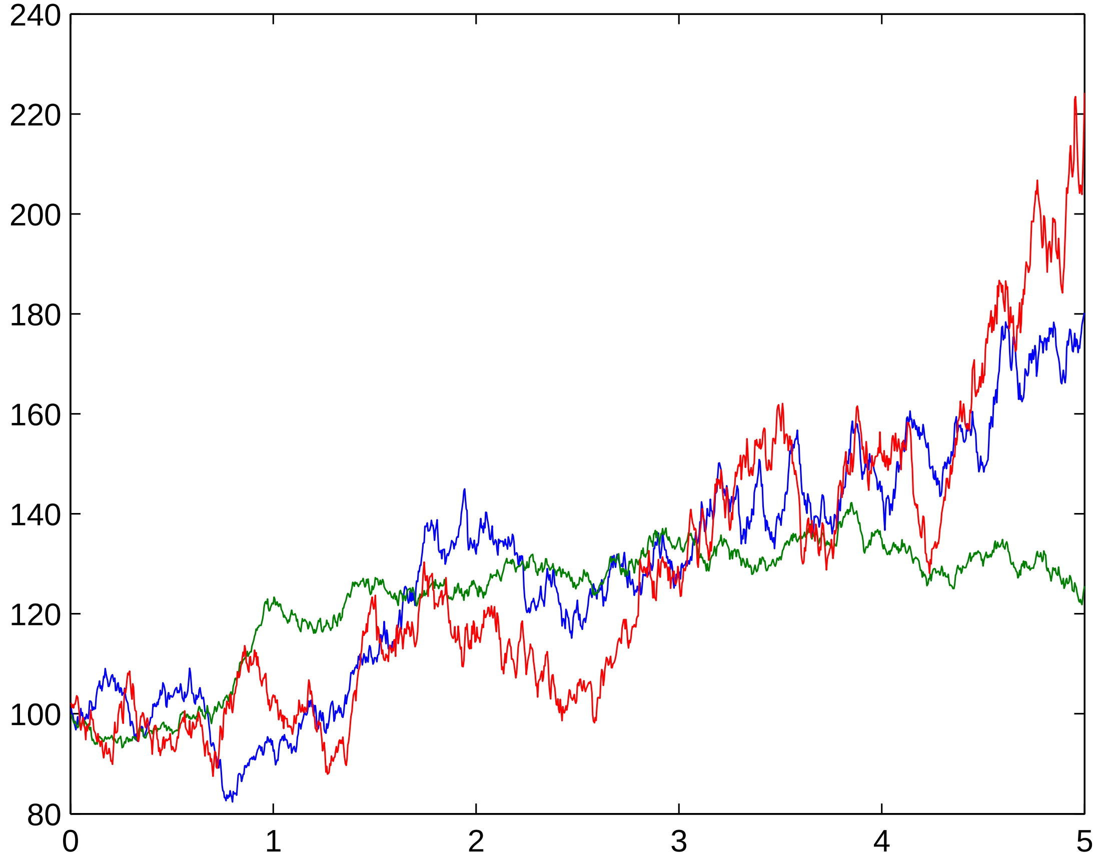


Figure 2: A joint evolution scenario for 3 asset classes.

Covariance of fundamentals:

$$\Sigma = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.3 \end{pmatrix}$$

Realized correlations:

$$\hat{C} = \begin{pmatrix} 1 & 20\% & 21\% \\ 20\% & 1 & 52\% \\ 21\% & 52\% & 1 \end{pmatrix}$$

Simulation of a 5- year scenario for 3 asset classes: realized vs fundamental correlations: $\lambda = 0.5$.

Zero fundamental correlation, significant realized correlation: effect entirely due to price impact of trading.

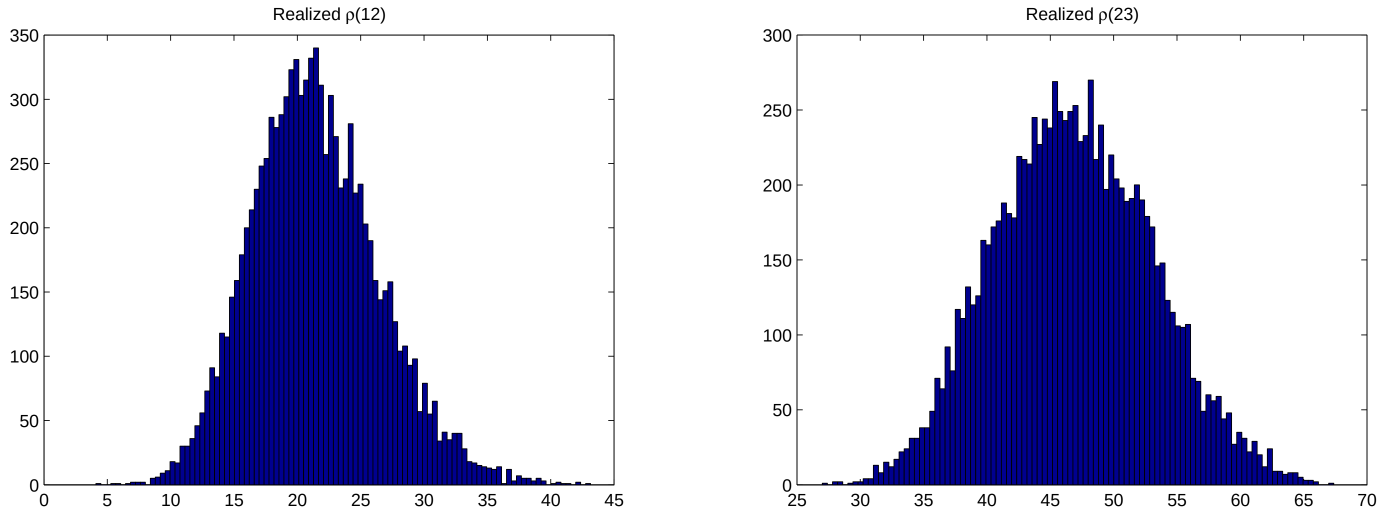


Figure 3: The distribution of realized correlations for different pairs of assets. Fundamental correlation is zero.

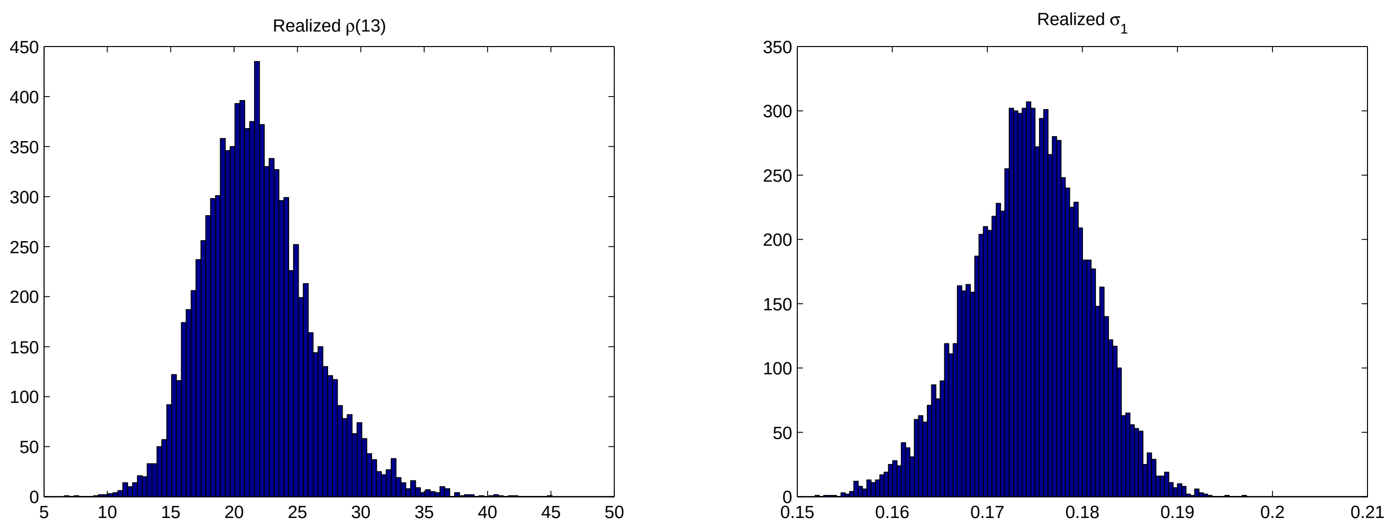


Figure 4: The distribution of realized correlations compared with the distribution of realized volatility.

Main findings

Presence of funds with (medium term) target allocations leads to a systematic component of demand for assets (sell when price rise, buy when prices fall).

The price impact of such strategies leads to *medium term mean reversion* in prices and realized correlations which are large and significant even if input/fundamental correlations are zero.

Realized correlation is path-dependent: it strongly varies across scenarios.

The distribution of realized correlation depends on the market depth/liquidity.

In statistical terms: realized correlations are strongly "biased" estimators.

Here "medium term"= horizon of allocation (typically 6 m-1 yr).

Diffusion approximation of Markov chains

Let (Y^n) is a sequence of $\mathbb{R}^d$-valued Markov chains $(Y^n_k)_{k\geq 0}$ with transition kernels $p_n(x,dy)$ and

$$b_n(x) \qquad = n\int_{|x-y|\leq 1}(y-x)p_n(x,dy) \tag{7}$$

$$a_n(x) \quad = n\int_{|x-y|\leq 1}(y-x)^t(y-x)p_n(x,dy) \tag{8}$$

If there exists $a \in C_0(\mathbb{R}^d, Sym(d\times d)_+), b \in C_0(\mathbb{R}^d, \mathbb{R}^d)$ such that

$$\sup_{|x|\leq R} |b_n(x) - b(x)| \to 0 \qquad \sup_{|x|\leq R} |a_n(x) - a(x)| \to 0 \tag{9}$$

$$a_n(x) \quad = n\int_{|x-y|\leq 1}(y-x)^t(y-x)p_n(x,dy) \tag{10}$$

and for $T > 0$ the weak solution to the SDE on $[0,\infty[$

$$dX_t = b(X_t)dt + a(X_t)dW_t \tag{11}$$

is unique on $[0,T]$ then $(X^n_k) = (Y^n_{[kn]})$ converges in distribution to X.

Proposition 1 (Continuous-time limit)**.** *Consider the above model with an "aggregate" fund with allocation* $X = (x_i, i = 1..n)$*.*
As the time step $h \to 0$*, the Markov chain* (S^h, Φ^h) *converges weakly to a diffusion limit*

$$(S^h, \Phi^h) \Rightarrow (S, \Phi)$$

where

$$\frac{dS_i(t)}{S_i(t)} = \mu_i dt + \frac{x_i V_t}{\lambda_i S_i(t)}\Big(\big(X'\mu - \mu_i\big)dt$$
$$+[1 - \frac{x_i V_t}{\lambda_i S_i(t)}\big(\big(X'\mu - \mu_i\big)](AdW)_i + X' AdW_t$$

$$\phi_i(t) = x_i \frac{V_t}{S_i(t)} \qquad \text{with } dV_t = V_t \sum_{i=1}^{n} \frac{X_i}{S_i(t)} dS_i(t)$$

where $A = \Sigma^{1/2}$ *is a positive square root of the (fundamental) covariance matrix and* V_t *is the value of the aggregate fund portfolio.*

Realized covariance The realized covariance of the returns is given by

$$\gamma_t^{ij} = \mathbf{\Sigma_{i,j}} + c_t^i\left[(\Sigma X)_j - \Sigma_{i,j}\right] + +c_t^j \left[(\Sigma X)_i - \Sigma_{i,j}\right]$$
$$+ c_t^i c_t^j \left[X'\Sigma X + \Sigma_{i,j} - (\Sigma X)_i - (\Sigma X)_j\right] \quad \text{where} c_t^i \quad = \frac{\Phi_t^i}{\lambda_i} = \frac{x_i V_t}{\lambda_i S_t^i}$$

Σ : fundamental covariance matrix, X: target allocation of fund

Even when fundamental correlations are ZERO, realized correlation is positive.

The level of observed correlation is INVERSELY PROPORTIONAL to market depth/liquidity.

Realized covariance: case of zero fundamental correlations

Proposition 2. *If Σ is diagonal then the realized covariance of the log returns is given by*

$$[\ln S_i, \ln S_j]_t = \int_0^t \gamma_{ij}(u)du \qquad \text{where} \tag{12}$$

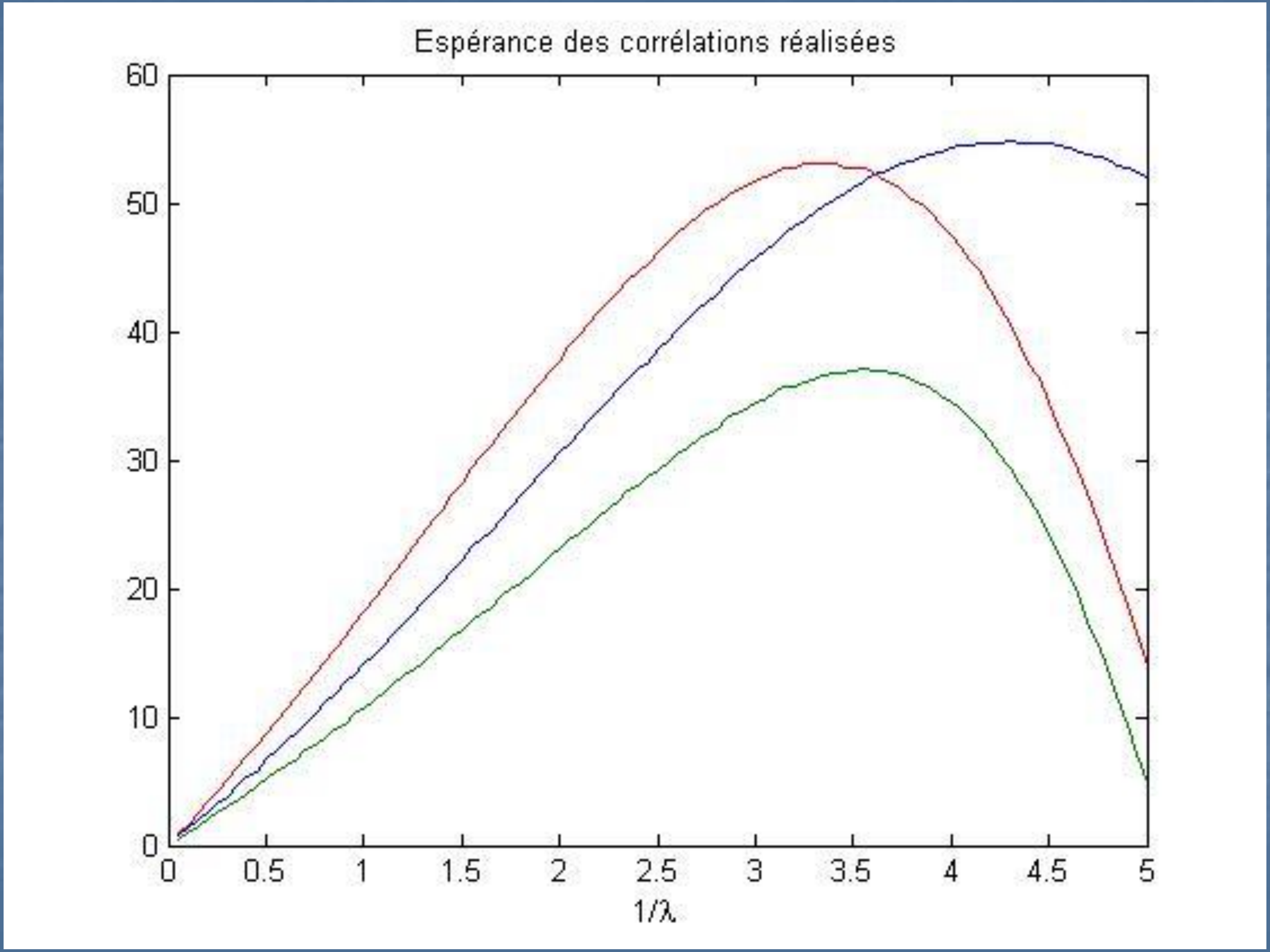
$$\gamma_t^{ij} = x_i x_j V_t \left[\frac{\sigma_j^2}{\lambda_i S_i(t)} + \frac{\sigma_i^2}{\lambda_j S_j(t)}\right] + \frac{x_i x_j V_t^2}{S_i(t) S_j(t)} \left[\sum_{k=1}^d x_k \sigma_k^2 - x_j \sigma_j^2 - x_i \sigma_i^2\right]$$

The realized variances are slightly lower due to liquidity effects

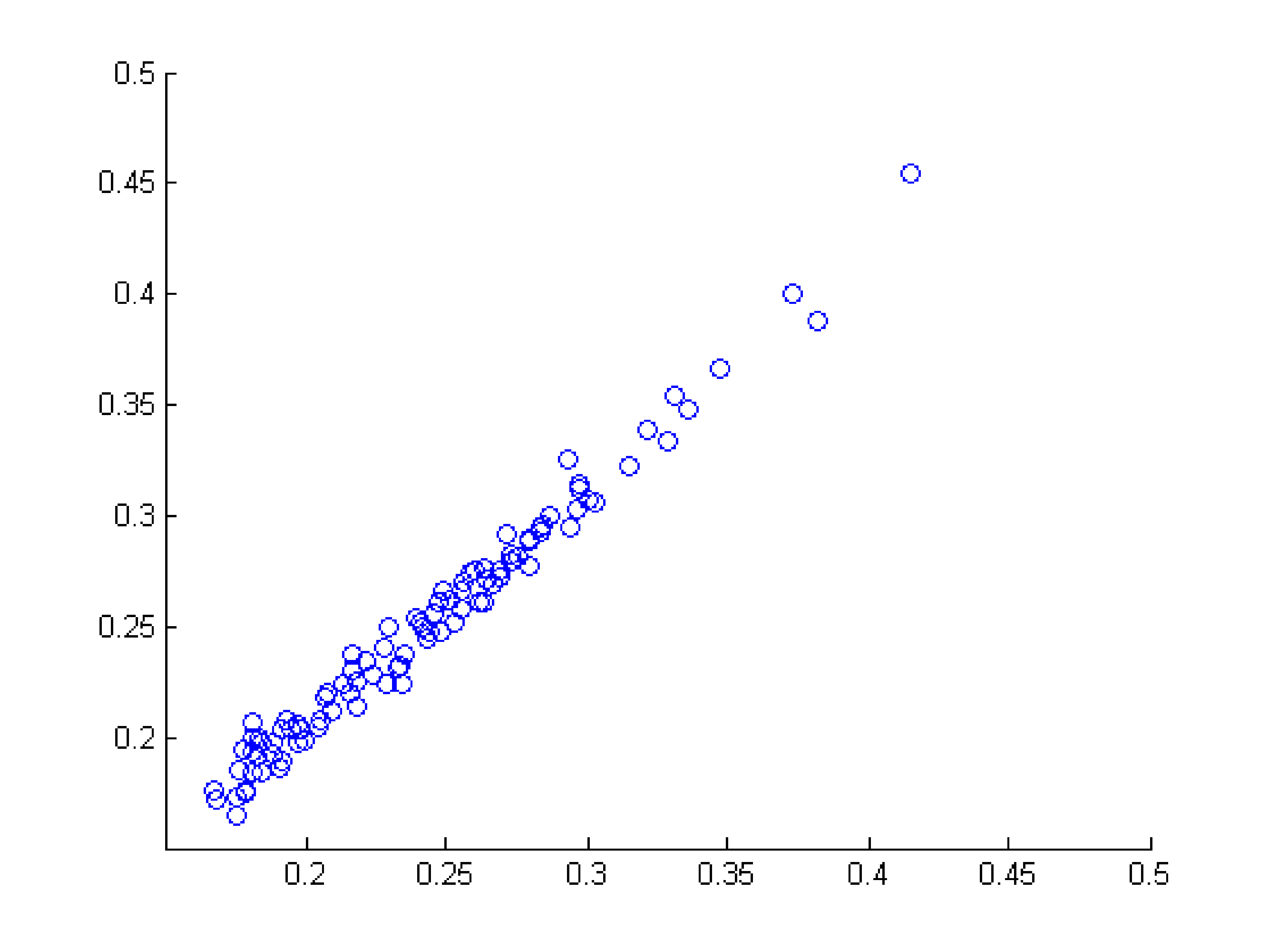
$$\gamma_t^{ii} = \sigma_i^2 - 2\frac{x_i(1-x_i)\sigma_i^2}{\lambda_i}\frac{V_t}{S_i(t)} + \frac{x_i^2}{\lambda_i^2}\frac{V_t^2}{S_i(t)^2}\left[\sum_{k=1}^d x_k \sigma_k^2 - (1-2x_i)\sigma_i^2\right]$$

Good agreement with simulated values even for $\delta t = 1$ week.

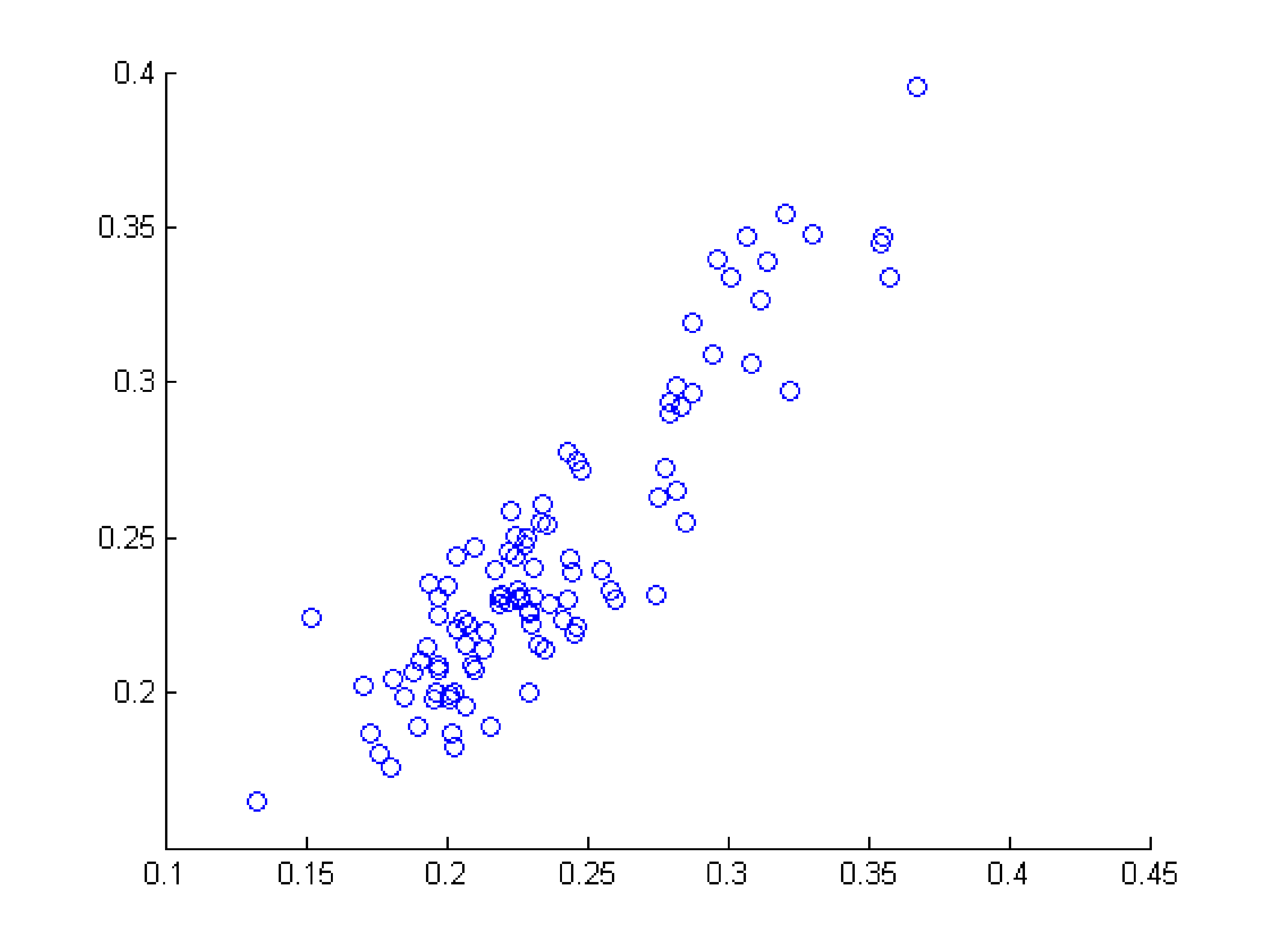
Variation of average level of realized correlation with market liquidity



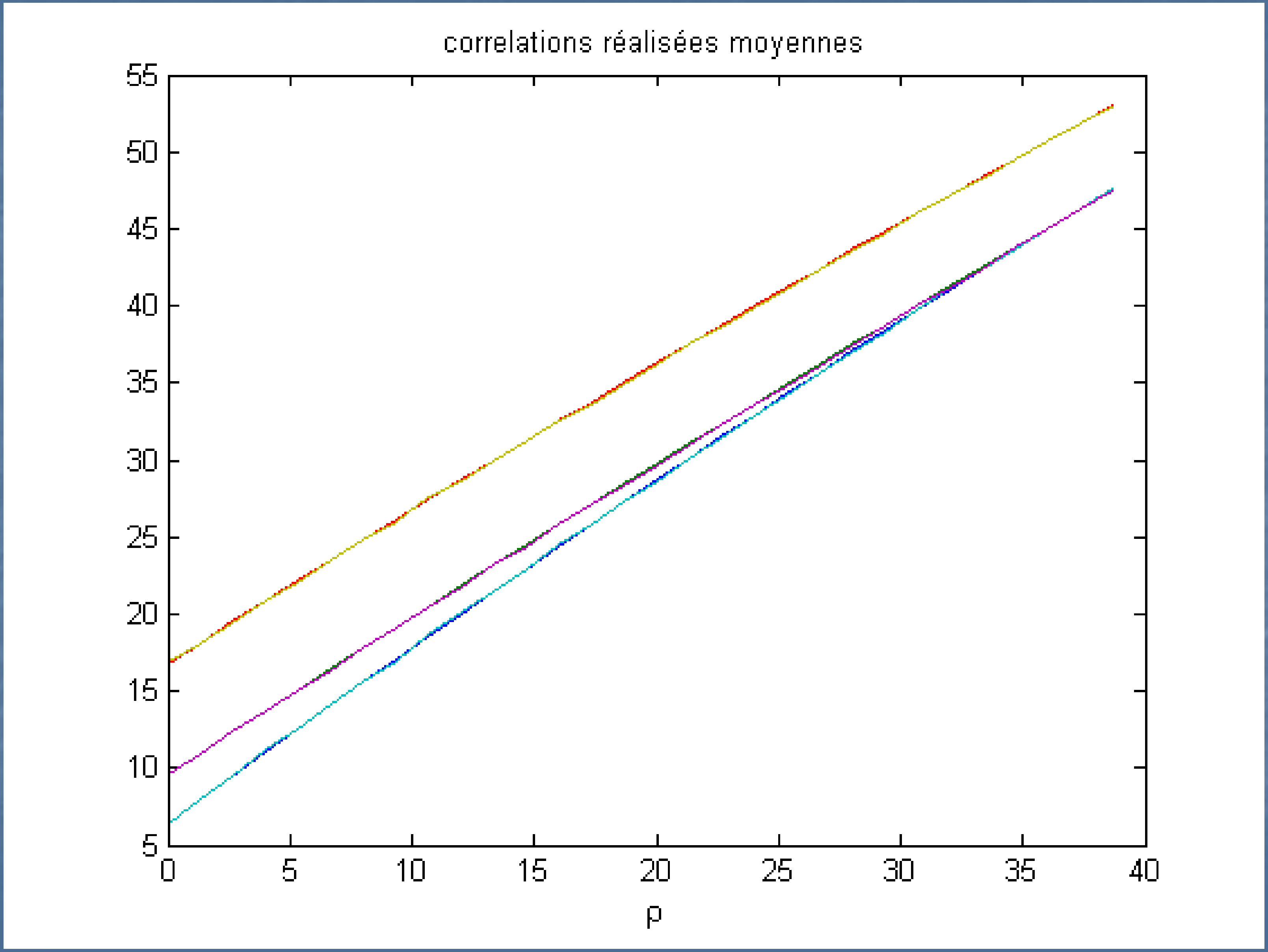
Realized correlations: simulation vs continuous-time asymptotics



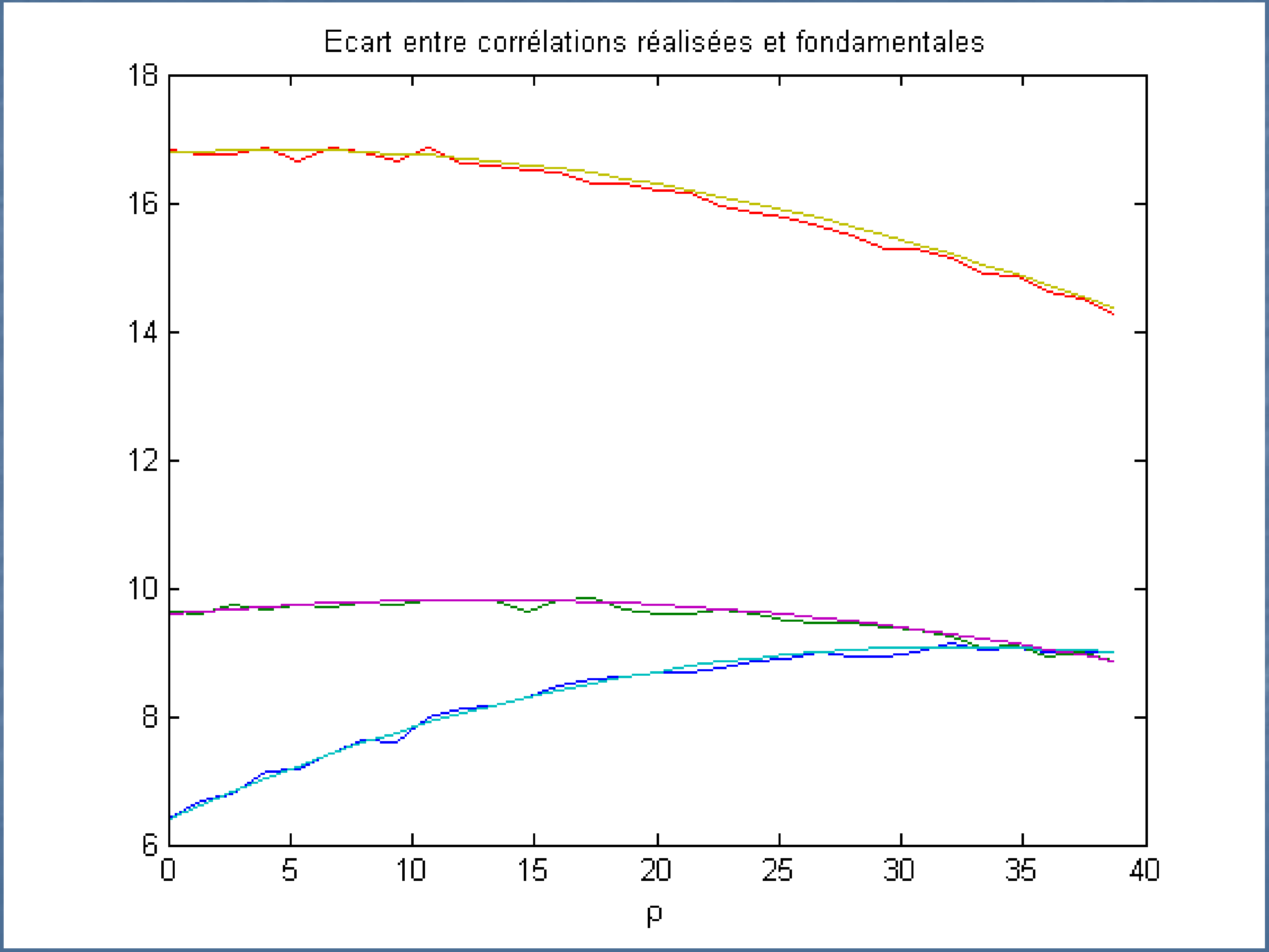
Realized correlations: simulation vs continuous-time asymptotics



Level of realized correlations as a function of fundamental correlations



Excess correlation: realized correlations minus fundamental correlations



Implementation

To implement the model and estimate the impact of trading on correlations we need

-an estimate of market depth/market impact: various econometric studies (see e.g. Wang & Obizhaeva,..)

-an estimate of the market capitalization of institutional investors in the market of interest: e.g. MSCI Emerging Market Funds

-the aggregate portfolio of these institutional investors: composition in asset classes of these indices

Second order effects: heterogeneity of allocations $\rightarrow$ *distribution* of allocations across funds : difficult to estimate

Example 2: running for the exit

Consider a multi-strategy fund investing in (say, two) strategies with *fundamentally uncorrelated* returns. Model the values of these strategies by two uncorrelated diffusions:

$$S^1_*(t + \Delta t) = S^1(t)[\mu_1 \Delta t + \sigma_1 \epsilon^1(t)] \tag{13}$$

$$S^2_*(t + h) = S^2(t)[\mu_2 \Delta t + \sigma_2 \epsilon^2(t)] \tag{14}$$

where $\epsilon^1(t), \epsilon^2(t)$ are independent.

The fund holds a leveraged long position in the two strategies S^1 and S^2.

$$V(t) = S^1(t) + S^2(t)$$

Running for the exit

When the value of the fund drops below a threshold K_1, some investors will exit the fund abruptly and other market participants might start trading against the fund by shorting it:

$$\phi(t) = c \max(K_1 - S^1(t) - S^2(t), 0) \tag{15}$$

What is the impact of exits and short selling on the volatility of the fund and the correlation between its strategies?

Modeling the effect of exiting/shorting a reference fund

Consider a leveraged fund whose holding in asset class i is α_i.

Market value at time t_k: $V_k = \sum_{i=1}^{n} \alpha_i S^i_{k\Delta t}$.

- Due to exogenous economic factors, asset prices move from $S_{k\Delta t}$ to $S^*_{(k+1)\Delta t}$. Short sellers and investors adjust their positions according to the new fund value V^*_{k+1}, .

- *Supply/demand imbalance moves prices:* $(S^i_{(k+1)\Delta t})^* \;\to\; S^i_{(k+1)\Delta t}$.

$$
\begin{tikzcd}
t_k \arrow[d] & & & & t_{k+1} \arrow[d] \\
\boxed{S_k, V_k} \arrow[rr, "\text{exogenous}", "\text{economic factors}"'] & & \boxed{S^*_{(k+1)}, V^*_{k+1}} \arrow[rr, "\text{short selling}", "\text{distressed selling}"'] & & \boxed{S_{(k+1)}, V_{k+1}}
\end{tikzcd}
$$

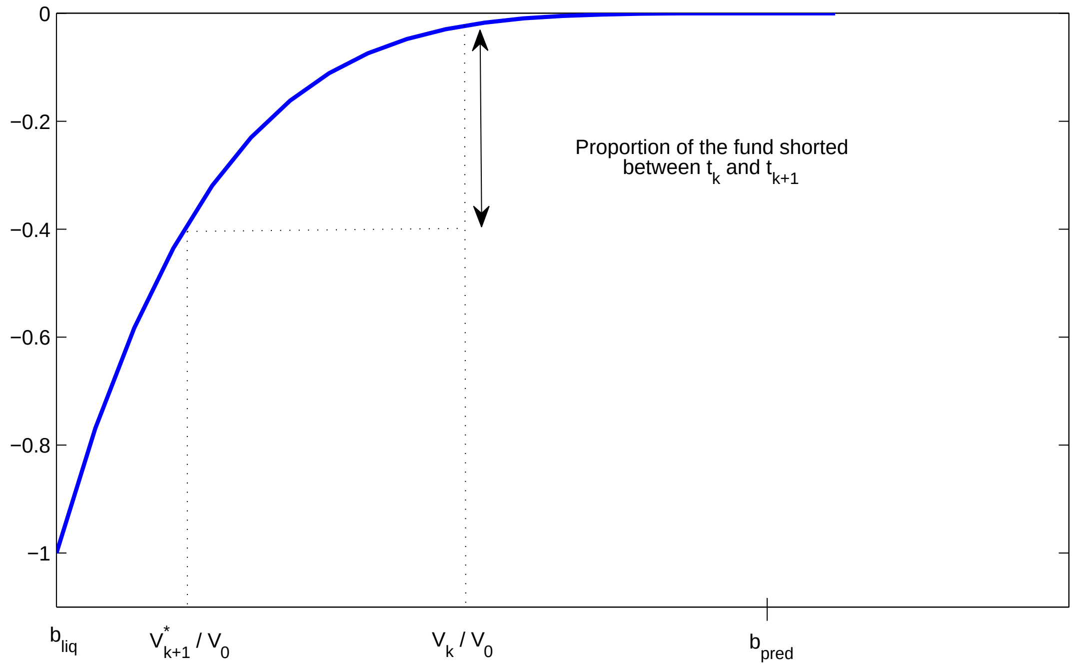


Figure 3: Demand generted by distressed sellers and short sellers as a function of fund value

The **net demand** for asset i between t_k and t_{k+1} is

$$-\alpha_i(f(\frac{V^*_{k+1}}{V_0}) - f(\frac{V_k}{V_0}))$$

where $f : \mathbb{R} \to \mathbb{R}$ is increasing, constant on $[\beta_{pred}, +\infty[$ and concave on $[\beta_{liq}, \beta_{pred}]$ with $0 \leq \beta_{liq} < \beta_{pred} \leq 1$ and α_i is the number of shares of i held by the fund.

- Distressed sellers do not trade when $V \geq \beta_{pred} V_0$.
- When the fund's market value goes below a $\beta_{liq} V_0$, short sellers and distressed sellers trade. If $V^*_{k+1} \leq V_k$, investors sell.
- If the fund value goes under a liquidation value $\beta_{liq} V_0$, it will face liquidation.
- When $\frac{V}{V_0} \in [\beta_{liq}, \beta_{pred}]$, the lower the fund value, the more investors selling, as f is concave on this interval.

Multi-period model model

$$\frac{S^i_{(k+1)\Delta t} - S^i_{k\Delta t}}{S^i_{k\Delta t}} = \sqrt{\Delta t}\ \epsilon^i_{k+1} + \frac{\alpha_i}{\lambda_i}(f(\frac{V^*_{k+1}}{V_0}) - f(\frac{V_k}{V_0}))$$

$S = S^{(\tau)} = (S^1, ..., S^n)$ is a Markov Chain.

Not easy to study analytically, but easy to simulate.

A simulated example with two asset classes

Consider a multi-strategy fund investing in two strategies with *fundamentally uncorrelated* returns. In absence of liquidity effects:

$$S^1_*(t+h) = S^1(t)[\mu_1 h + \sigma_1 \epsilon^1(t)] \tag{8}$$

$$S^2_*(t+h) = S^2(t)[\mu_2 h + \sigma_2 \epsilon^2(t)] \tag{9}$$

where $\epsilon^1(t), \epsilon^2(t)$ are IID and independent. The fund holds a leveraged long position S^1 and S^2.

$$V(t) = S^1(t) + S^2(t)$$

In absence of price impact/ liquidity effects: $S^i(t+h) = S^i_*(t)$

Simulated example: $c = 0.3, \lambda_1 = 100, \lambda_2 = 50,\ \mu_1 = \mu_2 = 4\%$

Initial leverage$=$ 10,
$K_1 = 0.975(S_1(0) + S_2(0)) > K_0 = 0.9(S_1(0) + S_2(0))$.

Price impact increase the default probability by a factor 30 and the correlation by 10 %.

	No price impact	With price impact
Sharpe ratio	1.1	0.75
Default probability	0.01%	0.3%
Correlation	0	11%
Conditional correlation in default scenario	0	37%

Table 1: Impact of liquidity on returns, Sharpe ratio and correlation between trades.

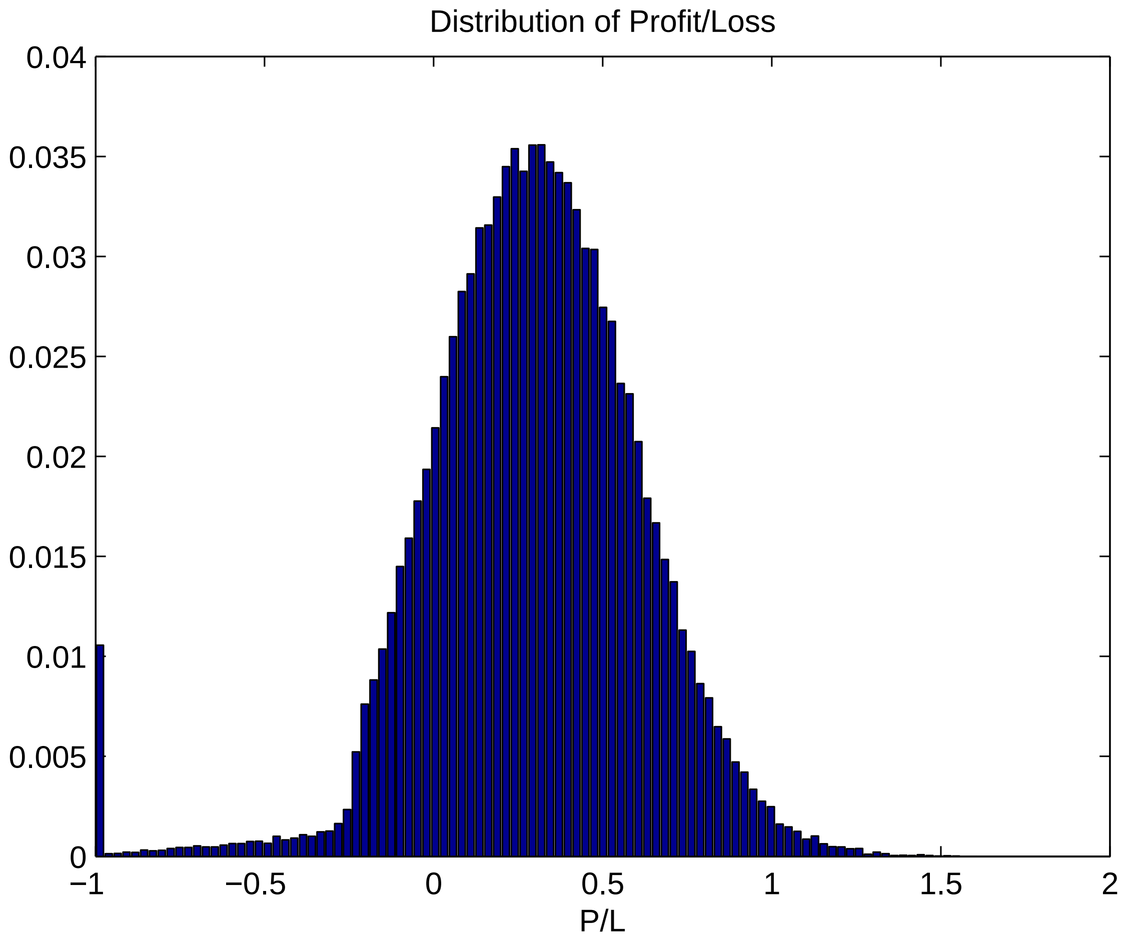


Figure 4: Profit/Loss distribution of fund as percentage of initial capital. $\rho = 0$, Leverage=10

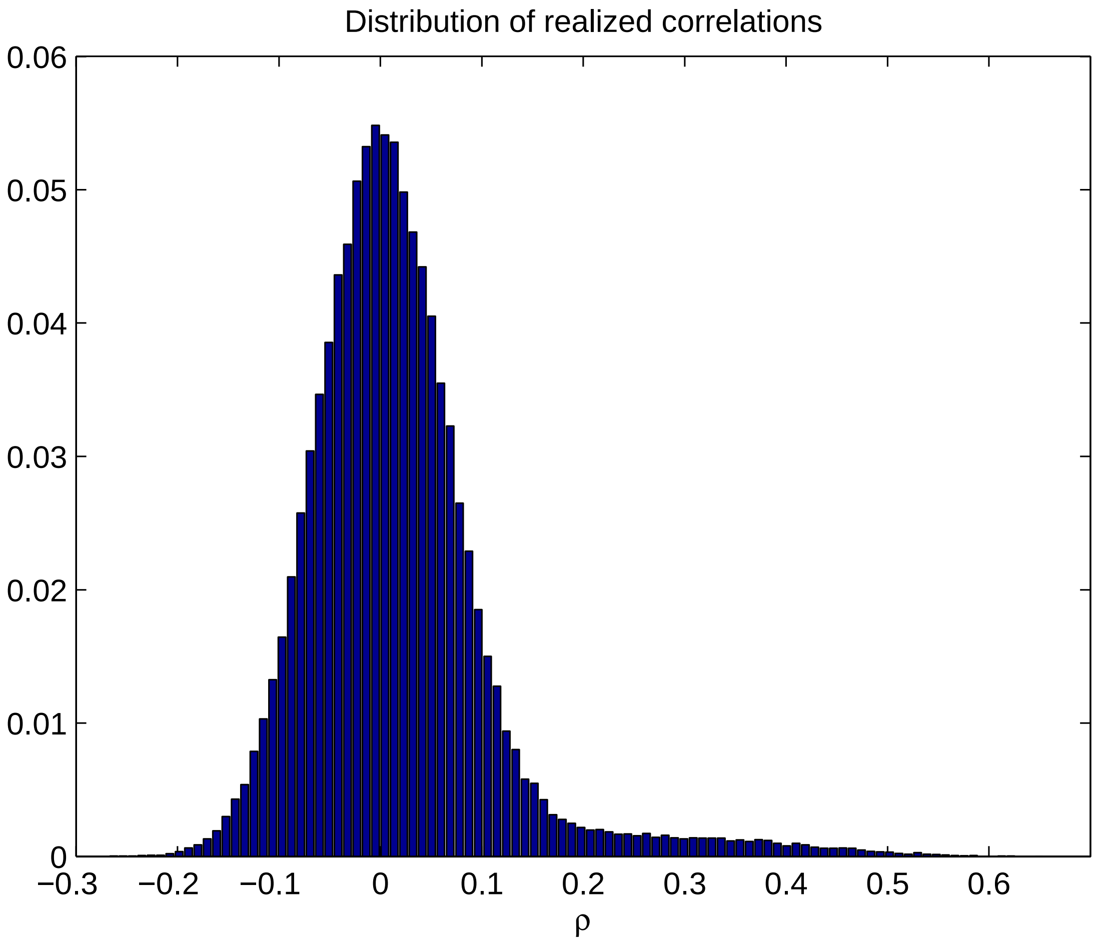


Figure 5: Distribution of one-year realized correlations. $\rho = 0$, Leverage=10

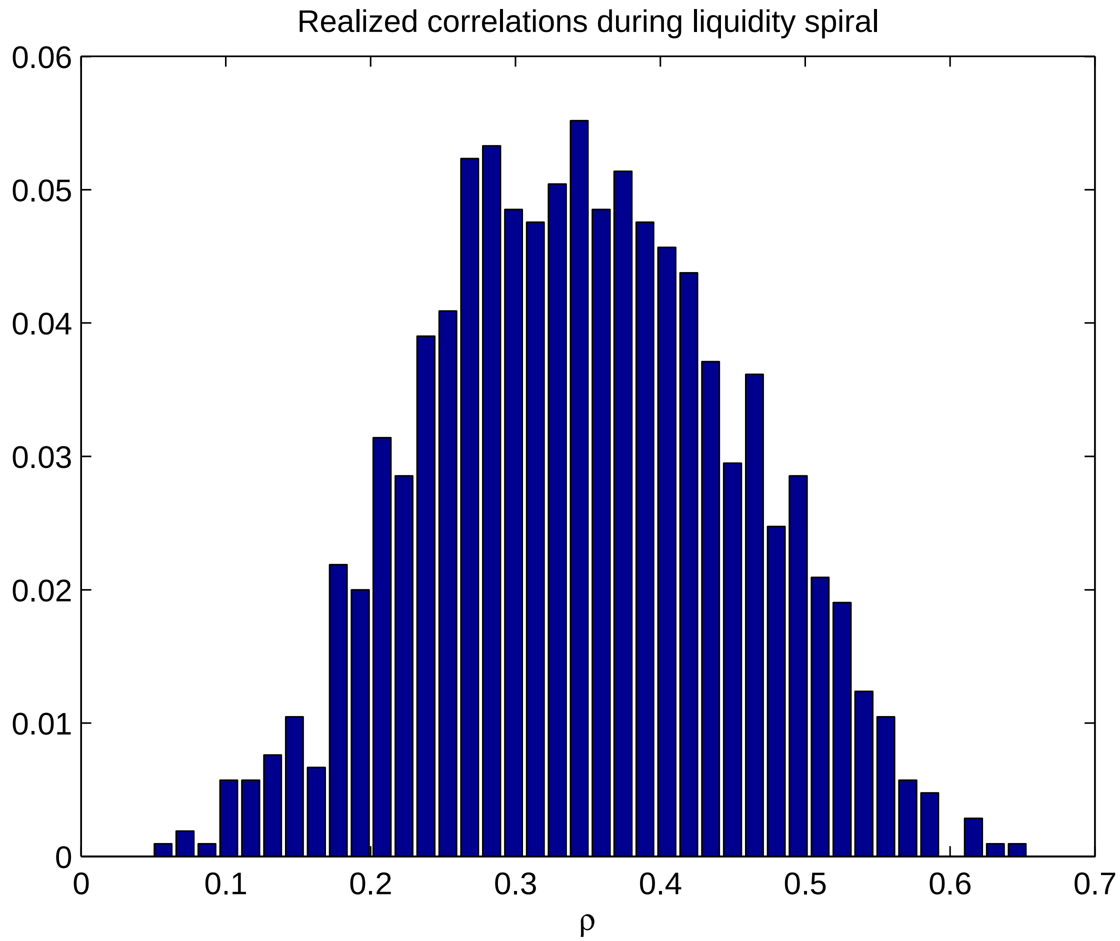


Figure 6: Distribution of one-year realized correlations conditional on default. $\rho = 0$

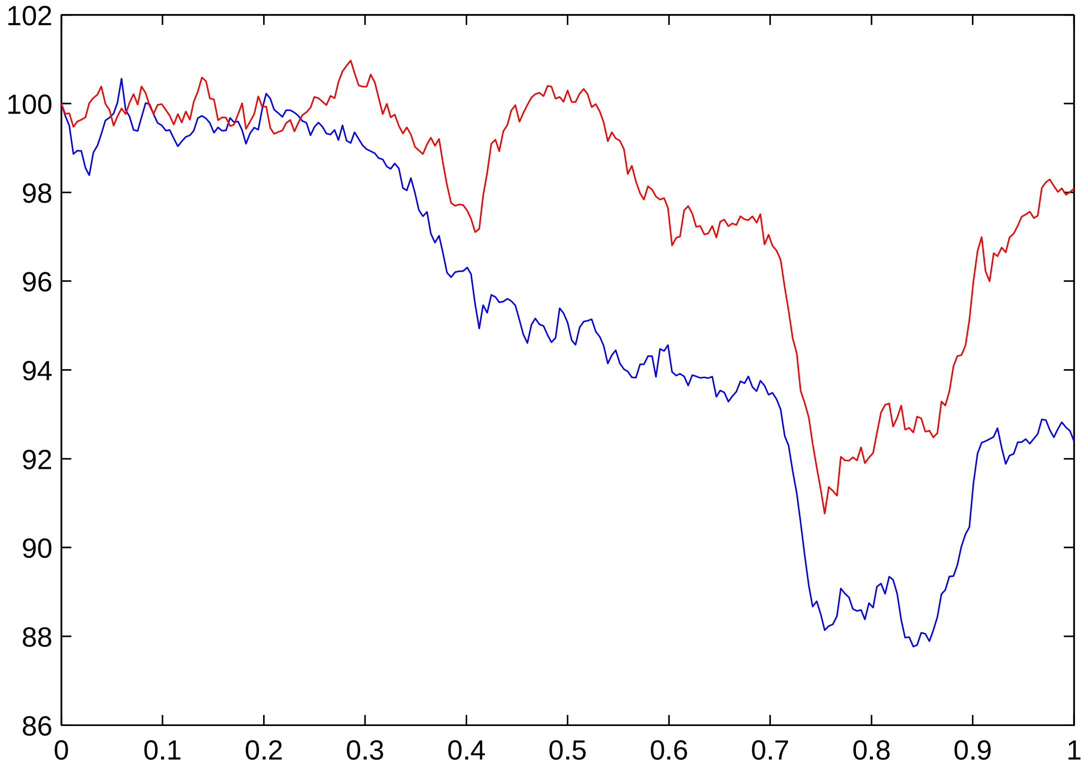


Figure 7: Example of a spiral to default. $\rho = 0$

Spiral to default: the “LTCM” effect

Bad performance of the fund is amplified by price impact of market players trading against it.

This not only drives down the price of the fund but drives up the correlation between the two strategies to extremely high levels.

As a result, realized correlation among strategies will be much higher than the “fundamental” correlation, exactly when the fund is in dire need of diversification.

The spiral can be triggered by a large loss in *one* of strategies. This leads to investors exiting the fund, others shorting its positions and thus generates a high correlation among *all* its positions.

Diffusion limit

Proposition 1. *Assume* $f \in C_b^3$ *and* $\mathbb{E}(|\epsilon_t^i|^4) < \infty$. *Then* $S_{t\geq 0}^{\Delta t}$ *converges weakly towards a diffusion* $P_t = (P_t^1, ...P_t^n)^t$ *when* $\Delta t \to 0$*:*

$$\frac{dP_t^i}{P_t^i} = (\mu_t)_i dt + (\sigma_t dW_t)_i \tag{13}$$

$$(\mu_t)_i = \frac{\alpha_i}{2\lambda_i} f^{''}(\frac{V_t}{V_0}) \frac{(\pi_t)^t \Sigma \pi_t}{V_0^2} \tag{14}$$

$$(\sigma_t)_{i,j} = A_{i,j} + \frac{\alpha_i}{\lambda_i} f^{'}(\frac{V_t}{V_0}) \frac{(A^t \pi_t)_j}{V_0} \tag{15}$$

with:

- $\pi_t = (\alpha_1 P_t^1, ..., \alpha_n P_t^n)^t$
- $V_t = \sum_{1\leq k\leq n} \alpha_k P_t^k$
- $AA^t = \Sigma$, *the fundamental covariance matrix*

Realized covariance

Instantaneous covariance between assets i and j has an additive decomposition: it is the sum of the fundamental covariance between i and j and terms of excess covariance

Proposition 2. *The realized covariance between securities i and j between 0 and t is equal to $\frac{1}{t}\int_0^t C_s^{i,j}\,\mathrm{d}s$, where $C_s^{i,j}$, the instantaneous covariance between i and j, is given by:*

$$C_s^{i,j} = \Sigma_{i,j} + \frac{\alpha_j}{\lambda_j} f^{'}(\frac{V_s}{V_0})\frac{(\Sigma\pi_t)_i}{V_0} + \frac{\alpha_i}{\lambda_i} f^{'}(\frac{V_s}{V_0})\frac{(\Sigma\pi_t)_j}{V_0}$$
$$+\frac{\alpha_i\alpha_j}{\lambda_i\lambda_j}(f^{'})^2(\frac{V_s}{V_0})\frac{((\pi_t)^t\Sigma\pi_t)}{V_0^2}$$

with

$$\pi_t = (\alpha_1 P_t^1, ..., \alpha_n P_t^n)^t$$

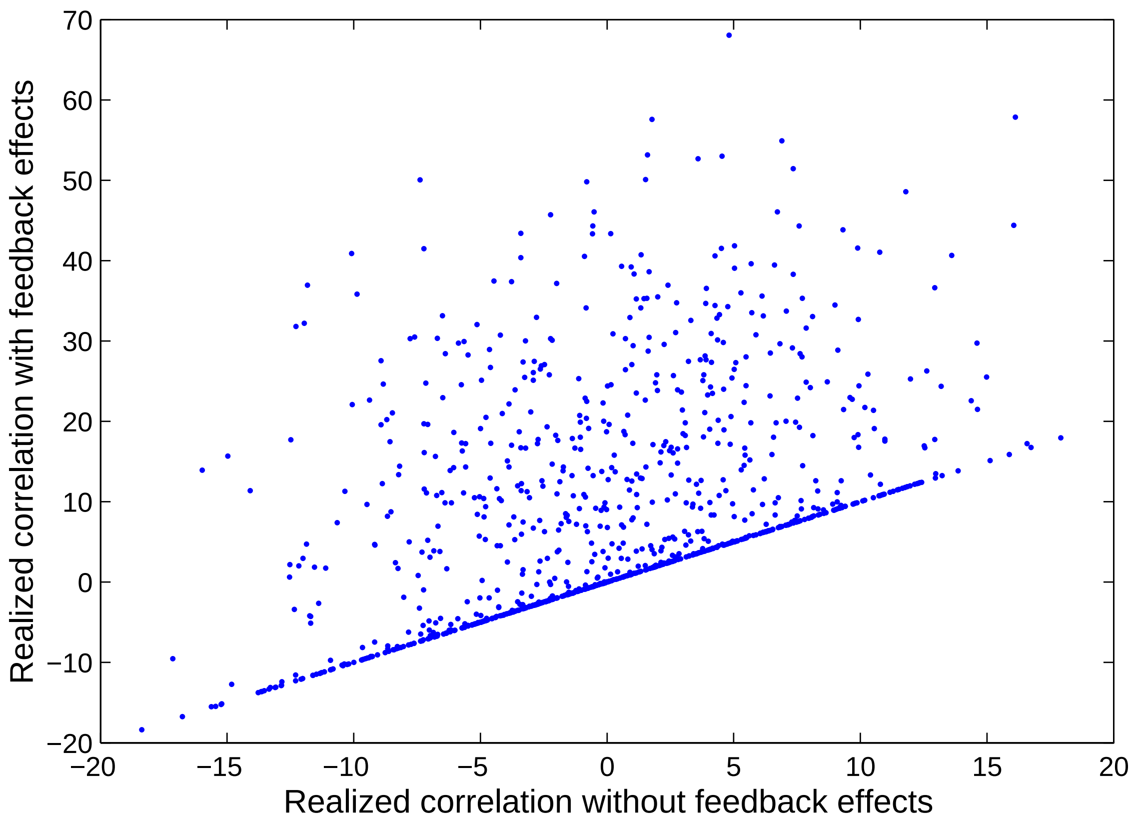


Figure 8: Realized correlation with and without feedback effects due to distressed selling.

Realized covariance: case of zero fundamental correlations

If the fundamental covariance matrix Σ is diagonal, then, for all $1 \leq i, j \leq n$, the instantaneous covariance between i and j $(i \neq j)$ and the instantaneous variance of asset i are given by:

$$C_t^{i,j} = \frac{\alpha_j}{\lambda_j} f^{'}(\frac{V_t}{V_0})\frac{\alpha_i}{V_0} P_t^i \sigma_i^2 + \frac{\alpha_i}{\lambda_i} f^{'}(\frac{V_t}{V_0})\frac{\alpha_j}{V_0} P_t^j \sigma_j^2 + \frac{\alpha_i \alpha_j}{\lambda_i \lambda_j}(f^{'})^2(\frac{V_t}{V_0}) \sum_{1 \leq l \leq n} (\frac{\alpha_l}{V_0} P_t^l \sigma_l)^2 > 0$$

and

$$C_t^{i,i} = \sigma_i^2 + 2\frac{\alpha_i}{\lambda_i} f^{'}(\frac{V_t}{V_0})\frac{\alpha_i}{V_0} P_t^i \sigma_i^2 + (\frac{\alpha_i}{\lambda_i})^2 (f^{'})^2 (\frac{V_t}{V_0}) \sum_{1 \leq l \leq n} (\frac{\alpha_l}{V_0} P_t^l \sigma_l)^2$$

Predatory trading/distressed selling creates positive correlation between assets with zero fundamental correlation

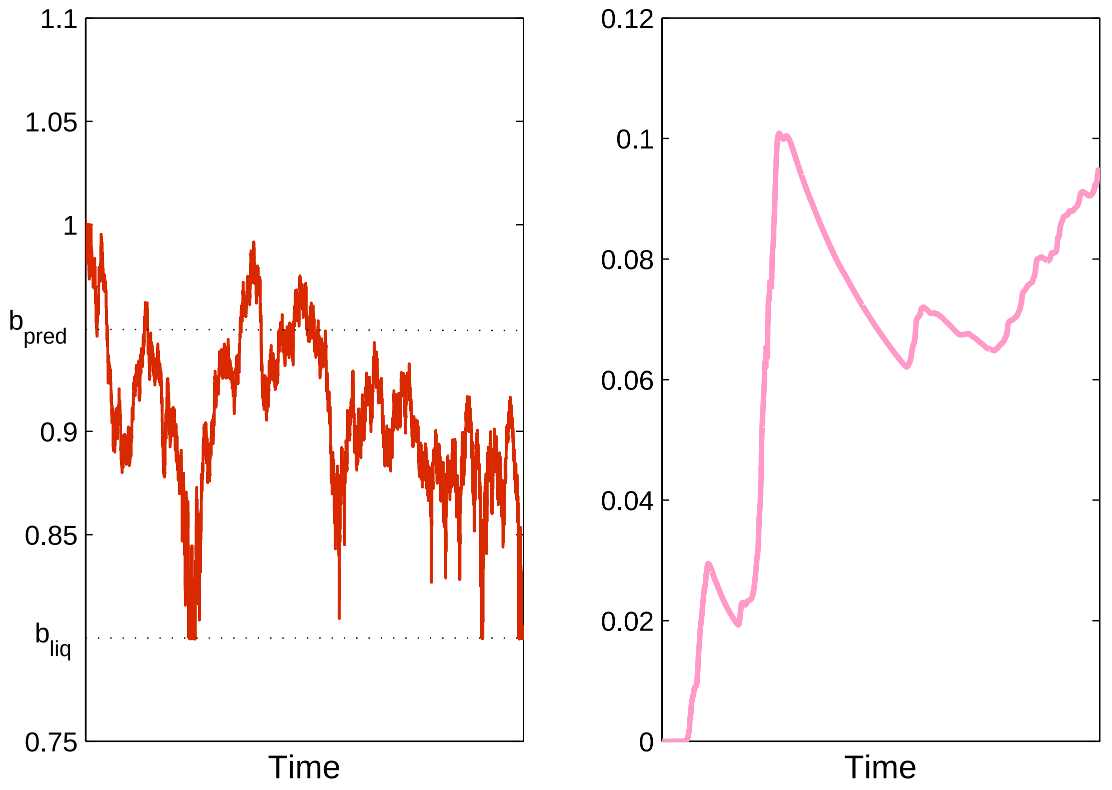


Figure 9: Left: value of reference fund. Right: realized correlation on $[0, t]$.

The role of liquidity

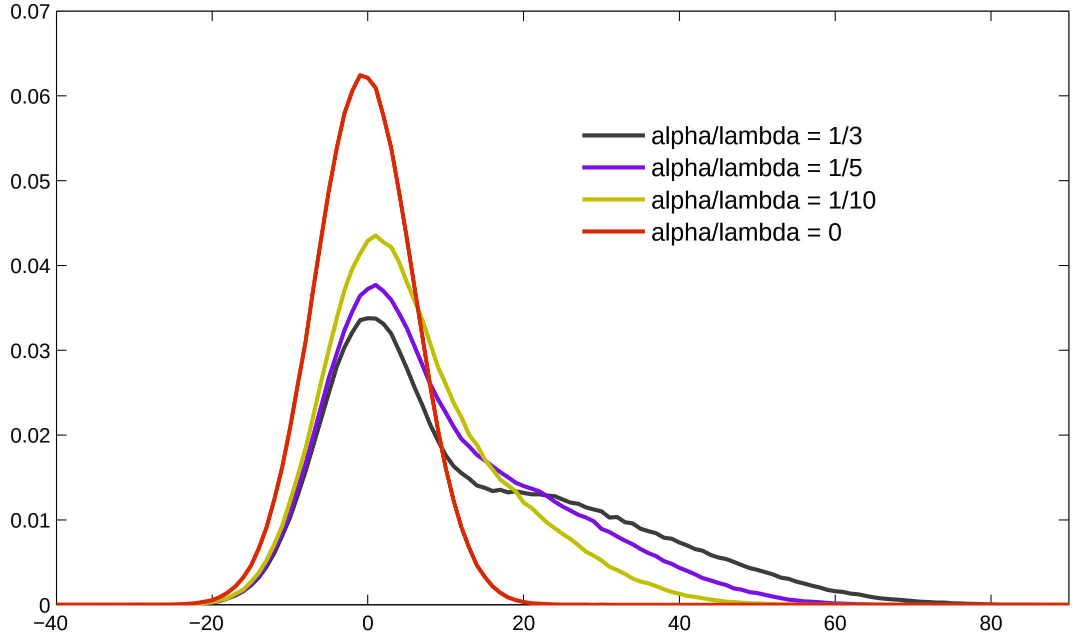


Figure 10: Distribution of realized correlation for different values of $\frac{\alpha}{\lambda}$

Impact on fund's realized variance: limits of diversification

Proposition 3. *The fund's realized variance between* 0 *and* t *is equal to* $\frac{1}{t}\int_0^t \Gamma_s \, \mathrm{d}s$ *where* Γ_s*, the instantaneous variance of the fund, is given by:*

$$\Gamma_s = \frac{1}{V_s^2}({}^t\pi_s \Sigma \pi_s + \frac{2}{V_0} f'(\frac{V_s}{V_0}) \left({}^t\Lambda \pi_s\right) \left({}^t\pi_s \Sigma \pi_s\right)$$
$$+\frac{1}{V_0^2} |f'(\frac{V_s}{V_0})|^2 \left({}^t\pi_s \Sigma \pi_s\right) |{}^t\Lambda \pi_s|^2)$$

with

$$\pi_t = (\alpha_1 P_t^1, ..., \alpha_n P_t^n)$$

and

$$\Lambda = (\frac{\alpha_1}{\lambda_1}, ..., \frac{\alpha_n}{\lambda_n})$$

Impact on volatility of other funds

Consider now a small fund with positions $(\mu_t^i, i = 1..n)$ and value $M_t = \sum_{1\leq i\leq n} \mu_t^i P_t^i$.

Proposition 4. *Under the assumption that M has negligible impact on market prices:*

$$d < M, M >_t = (\pi_t^\mu)^t \Sigma \pi_t^\mu + \frac{2}{V_0} f'(\frac{V_t}{V_0}) \left(\Lambda^t \pi_t^\mu\right) \left((\pi_t^\mu)^t \Sigma \pi_t^\alpha\right)$$
$$+ \frac{1}{V_0^2} (f'(\frac{V_t}{V_0}))^2 \left((\pi_t^\alpha)^t \Sigma \pi_t^\alpha\right) (\Lambda^t \pi_t^\mu)^2 .dt \qquad (16)$$

where

- $\pi_t^\alpha = (\alpha_1 P_t^1, ..., \alpha_n P_t^n)$
- $\pi_t^\mu = (\mu_t^1 P_t^1, ..., \mu_t^n P_t^n)$
- $\Lambda = (\frac{\alpha_1}{\lambda_1}, ..., \frac{\alpha_n}{\lambda_n})$.

Orthogonal funds: if $\sum_{1 \le i \le n} \frac{\alpha_i}{\lambda_i} \mu_t^i P_t^i = 0$, the fund M is UNAFFECTED by feedback effects created by short selling on the reference fund α.

whereas funds similar to α will experience large changes in realized correlation and volatility of their positions.

Example: August 2007 hedge fund crash

Investors exiting a large market-neutral long short fund can lead to high losses/excess volatility for *similar* long-short funds

But INDEX funds, being orthogonal to the reference fund, are unaffected.

This can happen WITHOUT liquidity drying up: even when λ_i are constant ($\neq$ explanation of Khandani & Lo)

Spiral to default: the "LTCM" effect

Bad performance of the fund is amplified by price impact of market players trading against it.

This not only drives down the price of the fund but drives up the correlation between the two strategies to extremely high levels.

As a result, realized correlation among strategies will be much higher than the "fundamental" correlation, exactly when the fund is in dire need of diversification.

The spiral can be triggered by a large loss in *one* of strategies. This leads to exits or adverse trading against the fund and generates a high correlation among *all* its positions.

Application: designing liquidity stress tests

These examples illustrate that a meaningful way to jointly stress correlation parameters is by introducing a liquidity term into the joint dynamics.

They provide a method for assessing the impact of a liquidity crisis on a portfolio: the model automatically adjust correlations in presence of a liquidity shock.

Example: what happens if liquidity is reduced in year 5?

Stress scenario: $\lambda = 1$ jumps to $\lambda = 0.25$ in year 5.

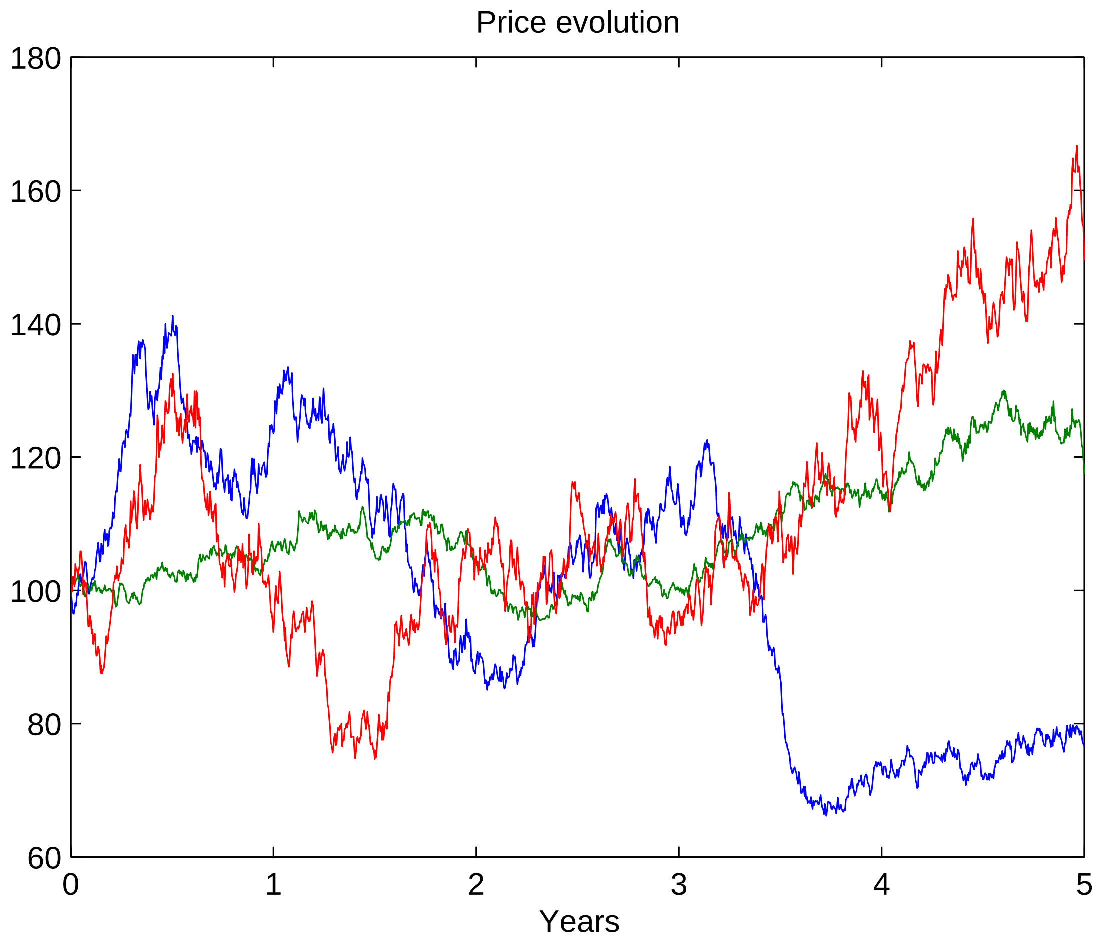


Figure 9: A liquidity crisis scenario (in year 5).

Covariance of fundamentals:

$$\Sigma = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.3 \end{pmatrix}$$

Realized correlations: before and during liquidity crisis

$$\hat{C}_{0-4} = \begin{pmatrix} 1 & 15\% & 12\% \\ 15\% & 1 & 25\% \\ 12\% & 25\% & 1 \end{pmatrix} \qquad \hat{C}_5 = \begin{pmatrix} 1 & 40\% & 50\% \\ 40\% & 1 & 33\% \\ 50\% & 33\% & 1 \end{pmatrix}$$

Simulation of a liquidity crisis for 3 asset classes: realized vs fundamental correlations: $\lambda = 1$ jumps to $\lambda = 0.25$ in year 5.

Zero fundamental correlation, significant realized correlation, correlation during liquidity crisis significantly different from "normal period".

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Some concluding remarks

- Simple stochastic models of joint dynamics of supply/demand and price provide useful insights into feedback effects and endogenous risk in financial markets.
- Endogenous risk has a significant predictable component which may be quantified using market observations.
- The incorporation of such risk pleads for the use of factors based on SIZE of strategies, not just returns.
- Such models show that
  - large moves in volatility and correlation cannot be separated from liquidity effects
  - **correlation risk** should be understood dynamically rather than in terms of static covariance matrices or copulas.
  - 'optimal strategies' may fail badly due to the endogenous patterns they generate themselves...