A Risk-based Model for the Valuation of Pension Insurance

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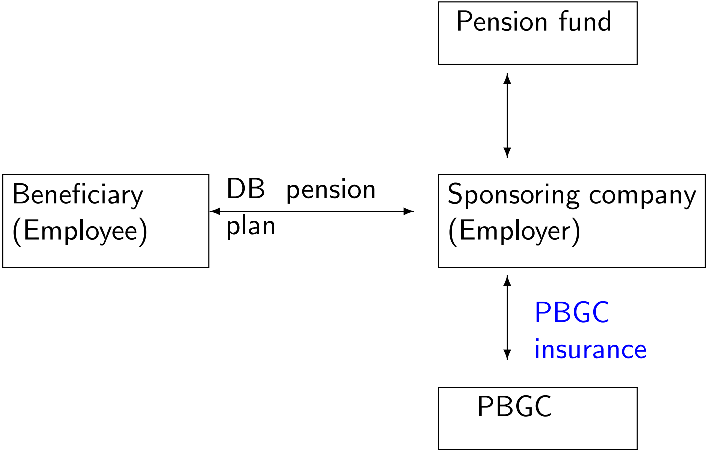
Pension Benefit Guaranty Corporation

- Pension Benefit Guaranty Corporation (PBGC):
  - ▷ established under the Employee Retirement Income Security Act of 1974
  - ▷ strengthening the retirement-income security of Americans
  - ▷ providing a federal insurance program to more than 44 million American workers and retirees in more than 27,500 defined benefit (DB) pension plans
    - ◊ DB plan: An employer-sponsored retirement plan where employee benefits are prespecified and based on an employee's salary and years of service .
    - ◊ Investment risk and portfolio management are entirely under the control of the sponsoring company
    - ◊ The sponsoring company is obliged to close insurance with PBGC


Pension fund
Beneficiary (Employee)
DB pension plan
Sponsoring company (Employer)
PBGC insurance
PBGC

Pension Benefit Guarantee Corporation

- The recent development of the PBGC
  - ▷ Some prominent cases for termination: Delphi, Lehman Brothers, Circuit City
- A variety of drivers
  - ▷ the recent sharp decline in the stock market beginning in 2000
  - ▷ general decline in the interest rate
  - ▷ ineffective premiums
  - ▷ ··· ···
- Focus of this paper: Risk-based premium calculation for the PBGC insurance

Premium calculation

- Prevailing premium calculation:
  - ▷ flat-rate premium
  - ▷ set by Congress and paid by sponsoring company
  - ▷ neglecting the investment policy
  - ⇒ Premiums not risk-based!
    - ▷ Sponsoring companies have a claim on their pension fund's surpluses (but limited liability)
    - ⇒ Sponsoring companies have an incentive to invest their pension funds in riskier assets, especially if the pension fund is insured.

⇒ A risked-based model for the PBGC insurance (contingent claim approach)

Existing literature: vanilla put option framework

- Contingent claim approach: pension insurance (PBGC support) modelled as a vanilla put option
  - ▷ The underlying asset of the put is the assets of the pension fund
  - ▷ The exercise price of the put is the promised pension payment
  - ▷ Literature: Sharpe (1976), Bodie and Merton (1993), and Marcus (1987), Chen, Ferris and Hsieh (1994), Lewis and Pennacchi (1994) and Bodie (1996), Boyce and Ippolito (2002)

This paper

This paper advocates a new approach to valuing the PBGC support by incorporating

- first, the support of the PBGC is a secondary guarantee provided to the pension funds
  - ▷ the sponsoring company provides a primal support to the pension plans
  - ▷ the sponsoring company covers the deficits of the pension fund when it does not lead to its own bankruptcy
- second, the pension fund can be prematurely terminated by an external regulator
  - ▷ premature termination is triggered by the underfunding of the pension fund;
  - ▷ premature termination means the pension fund is trusted by the PBGC

Main results

- We explicitly model the pension insurance provided by the PBGC taking account of the above two realistic aspects
- We are able to obtain an analytical valuation formula for the insurance premium
  - premium highly related to the funding status of the pension fund, the investment strategies of the sponsor and the pension fund
  - premium increasing in the correlation between the assets of the pension fund and of the sponsoring company

Agenda

- Introduction ($\surd$)
- Model setup
- Illustrative examples
- Conclusion

Defined benefit plan

- The pension plan is issued at time $t_0 = 0$ to a representative beneficiary
- The benefits are paid out as a lump-sum payment at the beneficiary's retirement date R: B_R
  - ▷ B_R is the prespecified benefit related to an employee's salary and years of service.
  - ▷ We assume that the pension liability B_R is deterministic.

Pension fund's asset process

- The pension fund invests in a constant equity-bond-mix
- The pension fund trades in the risk free bond F and the risky asset A in a self-financing way starting with initial wealth X_0

$$
\begin{aligned}
dX_t =& \theta \cdot X_t \cdot \frac{dA_t}{A_t} + (1-\theta) \cdot X_t \cdot \frac{dF_t}{F_t} \\
=& X_t(r + \theta(\mu - r)) + \theta\, \sigma\, X_t\, dW_t^1
\end{aligned}
$$

$$
\begin{aligned}
dA_t =& \mu A_t dt + \sigma A_t dW_t^1 \quad \text{(risky asset)} \\
dF_t =& r F_t dt \quad \text{(risk-free asset)}
\end{aligned}
$$

- ▷ θ is the fraction of wealth invested in the risky asset A and the remaining $(1-\theta)$ fraction invested in the risk free bond F
- ▷ W^1 is a standard Brownian motion under the market probability measure P

Sponsoring company's asset process

- The sponsoring company's assets also follow Black-Scholes dynamics with instantaneous rate of return $\mu_c > 0$ and volatility $\sigma_c > 0$

$$dC_t = \mu_c C_t dt + \sigma_c C_t(\rho dW_t^1 + \sqrt{1-\rho^2} dW_t^2)$$

  - ▷ W^2 is again a standard Brownian motion under the market measure P, independent of W^1.
  - ▷ The sponsoring corporation's and the pension fund's assets are correlated with a correlation coefficient $\rho \in [-1, 1]$.

Regulatory monitoring

- Assumption: an external pension regulator continuously monitors the pension fund's asset.
- The regulator closes the pension fund and gives it to the PBGC if at any $t \leq R$

$$X_t \leq \eta\, B_R\, e^{-r(R-t)}$$

  - ▷ The regulatory threshold: $\eta\, B_R\, e^{-r(R-t)}$ for time $t \leq R$. We assume $\eta \in (0,1)$
  - ▷ Let $\tau = \inf\{t | X_t \leq \eta\, B_R\, e^{-r(R-t)}\}$
- If the pension fund's assets over-perform the threshold all the time, the pension fund is terminated naturally at time R.

Payoff structure of the sponsor support

- The sponsor support depends on its own funding situation:
  - ▷ Default of the sponsoring company as the event that its assets are insufficient to pay back its own outstanding debt $\phi C_0 e^{gt}$
  - ▷ Providing the support shall not lead to its own default
    - ▷ when the sponsoring company is already defaulted $\Rightarrow$ no support
    - ▷ the sponsor's assets perform extremely well $\Rightarrow$ full support
    - ▷ the sponsor's assets perform moderately $\Rightarrow$ partial support
- Sponsor guarantees: $\Phi_c(\tau)$ (the payment sponsor makes at τ); $\Phi_c(R)$ (the payment sponsor makes at time R)

Sponsor support given premature termination $\Phi_c(\tau)$

- Deficit of the pension fund at τ: $B_R e^{-(R-\tau)} - X_\tau$
- The funding situation of the sponsoring company is examined at τ:
  a) Its assets are even insufficient to pay back its own outstanding debt $C_\tau < \phi C_0 e^{g\tau} \Rightarrow$ no support
  b) The company's assets perform extremely well, when $C_\tau \geq \phi C_0 e^{g\tau} + B_R e^{-r(R-\tau)} - X_\tau \Rightarrow$ full support $B_R e^{-r(R-\tau)} - X_\tau$
  c) The sponsoring company's assets perform moderately, when $\phi C_0 e^{g\tau} < C_\tau < \phi C_0 e^{g\tau} + B_R e^{-r(R-\tau)} - X_\tau \Rightarrow$ partial support $C_\tau - \phi C_0 e^{g\tau}$

Sponsor support at maturity date R ($\Phi_c(R)$)

- Deficit of the pension fund at R: $(B_R - X_R)1_{\{X_R<B_R\}}$

$$\Phi_c(R) = \begin{cases} 0 & C_R < C_0 e^{gR} \\ B_R - X_R & C_R > \phi C_0 e^{gR} + (B_R - X_R) \text{ and } X_R < B_R \\ C_R - \phi C_0 e^{gR} & \phi C_0 e^{gR} < C_R < \phi C_0 e^{gR} + (B_R - X_R) \text{ and } X_R < B_R \end{cases}$$

- We can express the entire support provided by the sponsoring company as:

$$\Phi_c = \Phi_c(\tau)1_{\{\tau\leq R\}} + \Phi_c(R)1_{\{\tau>R\}}.$$

Payoff profile of the PBGC insurance

- We assume here that the PBGC covers all the residual deficits that the sponsoring company is unable to cover:
  - ▷ Upon termination by the regulator, the insurance of the PBGC is

$$G(\tau) = (B_R e^{-r(R-\tau)} - X_\tau) - \Phi_c(\tau)$$

  - ▷ Upon natural termination, the insurance of the PBGC is

$$G(R) = (B_R - X_R)1_{\{X_R < B_R\}} - \Phi_c(R).$$

  - ▷ More compactly, the insurance of the PBGC

$$G = G(\tau)1_{\{\tau \leq R\}} + G(R)1_{\{\tau > R\}}.$$

Risk-neutral pricing

- The payoff profile for the PBGC insurance can be understood as exotic options ⇒ Risk-neutral pricing
  - ▷ One can first adjust the probability measure such that it incorporates the effect of risk, then takes the expected discounted value under the adjusted measure.
  - ▷ This new measure is the so-called risk-neutral probability measure P^* and the valuation method is risk-neutral pricing.

Market cost of the PBGC insurance

- The market cost (premium) paid by the sponsoring company to the PBGC is the expected discounted insurance payoff under the risk neutral probability measure:

$$G_0 = E^* \left[e^{-rR} G(R) 1_{\{\tau > R\}} \right] + E^* \left[e^{-r\tau} G(\tau) 1_{\{\tau \le R\}} \right]$$

  where E^* denotes the expected value under the risk-neutral measure P^*.

- We can also calculate "pseudo-premium" the sponsor would have obtained as a compensation for providing support

$$\Phi_c(0) = E^* \left[e^{-rR} \Phi_c(R) 1_{\{\tau > R\}} \right] + E^* \left[e^{-r\tau} \Phi_c(\tau) 1_{\{\tau \le R\}} \right]$$

- Closed-form solutions are obtained.

Way of solutions

Write down the assets processes under the risk-neutral measure

- Premium (premature termination)
  - ▷ At time τ, it holds

$$X_\tau = X_0 \exp\left\{\left(r - \frac{1}{2}\sigma^2\theta^2\right)\tau + \sigma\theta W_\tau^{1*}\right\} = \eta B_R e^{-r(R-\tau)} \Rightarrow W_\tau^{1*}$$

  - ▷ Substitute W_τ^{1*} in C_τ (depending on W_τ^{1*} and W_τ^{2*})
  - ▷ Transform the condition $C_0 e^{g\tau} < C_\tau < \phi C_0 e^{g\tau} + (B_\tau - X_\tau)$ to the condition $d_2(\tau) < W_\tau^{2*} < d_1(\tau)$
  - ▷ Use the density of the first-hitting τ
- Premium (natural termination)
  - ▷ Use the property of iterated expectations
  - ▷ We use the fact that $\ln X$ and $\ln C$ follow the cumulative bivariate normal distribution with correlation coefficient ρ.
  - ▷ Use the probability $P^*(\tau > R)$

Parameter choices

- For the numerical illustration, we fix the relevant parameters as follows:

$$\theta = 0.6,\ \phi = 0.6,\ r = 0.05,\ g = 0.02,\ \eta = 0.8,$$
$$\sigma = 0.20,\ \sigma_c = 0.3333,\ R = 15,\ X_0 = 100,\ B_R = 190.3$$

  - ▷ θ: weight in equity holding
  - ▷ ϕ: leverage ratio; g: growth rate of debt level
  - ▷ η: regulatory parameter
  - ▷ σ, σ_c: volatility of the pension fund and the company's assets

Premium for PBGC and sponsoring company

$\rho = -0.5$		$\rho = -0.25$		$\rho = 0$	
Premium		Premium		Premium	
Sponsor	PBGC	Sponsor	PBGC	Sponsor	PBGC
8.568	2.240	7.601	3.207	6.602	4.206

$\rho = 0.25$		$\rho = 0.5$	
Premium		Premium	
Sponsor	PBGC	Sponsor	PBGC
5.498	5.310	4.181	6.627

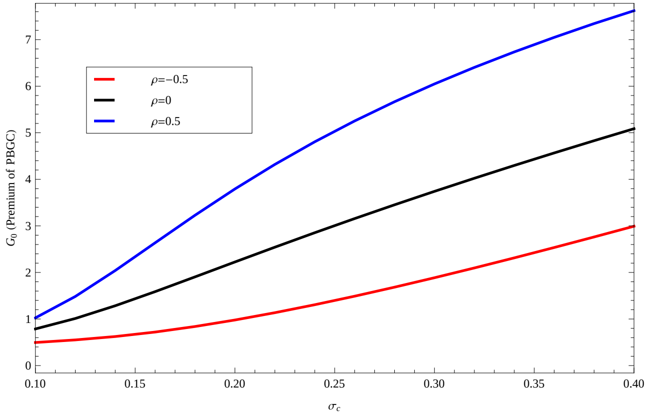
Implied premium for providing the sponsor guarantee and PBGC guarantee.

Risk-based premium

θ	$\eta = 0.8$			Vanilla put
	$\rho = -0.5$	$\rho = 0$	$\rho = 0.5$	
0.1	0.033	0.145	0.258	0.294
0.3	1.025	2.438	3.984	4.670
0.5	1.957	3.891	6.221	10.116
0.7	2.433	4.362	6.760	15.673

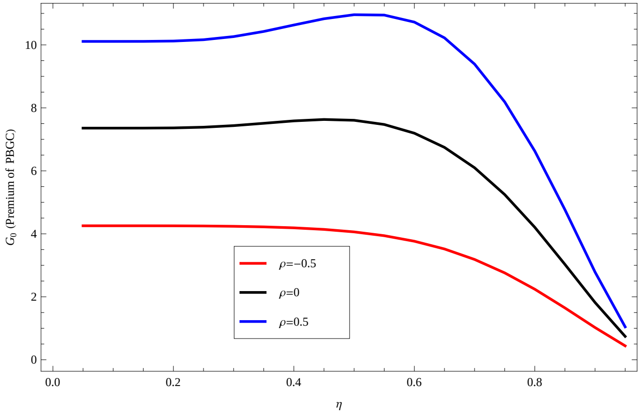
Fair premium for PBGC's insurance as a function of θ and ρ.

Premium as a function of vola of the company's asset σ_c



Fair premium G_0 as a function of the volatility of the company's assets σ_c.

Premium as a function of the regulatory parameter η



Summary

- We model the pension insurance provided by the PBGC and determine the risked-based premium for this insurance.
- The premium is highly related to the funding status of the pension fund, the investment strategies of the sponsor and the pension fund, and particularly the correlation between these two portfolios.
- Vanilla put can be considered as a limiting case and provides a price upper bound to our case

Premium responsible for premature default

$$
\begin{aligned}
& E^* \left[e^{-r\tau} G(\tau) 1_{\{\tau \leq R\}} \right] \\
= & \int_0^T e^{-rs} (1-\eta)\, B_R\, e^{-r(R-s)} N\left(\frac{d_2(s)}{\sqrt{s}} \right) f(s) ds \\
& + \int_0^T e^{-rs} \left((1-\eta) B_R e^{-r(R-s)} + \phi C_0 e^{gs} \right) \left(N\left(\frac{d_1(s)}{\sqrt{s}} \right) - N\left(\frac{d_2(s)}{\sqrt{s}} \right) \right) f(s) \\
& - \int_0^T e^{-rs} C_0 \exp\left\{ \left(r - \frac{1}{2}\sigma_c^2 \right) s + \sigma_c \rho \frac{\ln \frac{\eta B_R e^{-r(R-s)}}{X_0} - (r - \frac{1}{2}\theta^2\sigma^2) s}{\theta\sigma} \right\} \\
& \cdot \left(\int_{d_2(s)/\sqrt{s}}^{d_1(s)/\sqrt{s}} \exp\{\sigma_c \sqrt{1-\rho^2} \sqrt{s} x\} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \right) f(s) ds
\end{aligned}
$$

where $f(s)$ is the density of the first hitting time τ under P^*.

Premium responsible for maturity termination

$$
\begin{aligned}
& E^*\left[e^{-rR}G(R)1_{\{\tau>R\}}\right] \\
=& P^*(\tau>R)\int_{d_{x1}}^{d_{x2}}\int_{-\infty}^{d_{y1}} e^{-rR}\left(B_R - X_0\exp\left\{(r-\frac{1}{2}\theta^2\sigma^2)R+\theta\sigma\sqrt{R}x\right\}\right) \\
& \frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}(x^2+y^2-2\rho xy)\right\}dydx \\
& + P^*(\tau>R)\int_{d_{x1}}^{d_{x2}}\int_{d_{y1}}^{d_{y2}} e^{-rR}\Bigg(\left(B_R - X_0\exp\left\{(r-\frac{1}{2}\theta^2\sigma^2)R+\theta\sigma\sqrt{R}x\right\}\right) \\
& -\left(C_0\exp\{(r-\frac{1}{2}\sigma_c^2)R+\sigma_c\sqrt{R}y\}-\phi C_0e^{gR}\right)\Bigg) \\
& \frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}(x^2+y^2-2\rho xy)\right\}dydx
\end{aligned}
$$