A Risk-based Model for the Valuation of Pension Insurance

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Pension Benefit Guaranty Corporation

- Pension Benefit Guaranty Corporation (PBGC):
  - established under the Employee Retirement Income Security Act of 1974
  - strengthening the retirement-income security of Americans
  - providing a federal insurance program to more than 44 million American workers and retirees in more than 27,500 defined benefit (DB) pension plans
  - DB plan: An employer-sponsored retirement plan where employee benefits are prespecified and based on an employee’s salary and years of service.
  - Investment risk and portfolio management are entirely under the control of the sponsoring company
  - The sponsoring company is obliged to close insurance with PBGC
A Risk-based Model for the Valuation of Pension Insurance
The recent development of the PBGC

- Some prominent cases for termination: Delphi, Lehman Brothers, Circuit City

A variety of drivers

- the recent sharp decline in the stock market beginning in 2000
- general decline in the interest rate
- ineffective premiums
- ... ...

Focus of this paper: Risk-based premium calculation for the PBGC insurance
Premium calculation

- Prevailing premium calculation:
  - flat-rate premium
  - set by Congress and paid by sponsoring company
  - neglecting the investment policy
  => Premiums not risk-based!
  - Sponsoring companies have a claim on their pension fund’s surpluses (but limited liability)
  => Sponsoring companies have an incentive to invest their pension funds in riskier assets, especially if the pension fund is insured.
  => A risked-based model for the PBGC insurance (contingent claim approach)
Contingent claim approach: pension insurance (PBGC support) modelled as a vanilla put option

- The underlying asset of the put is the assets of the pension fund
- The exercise price of the put is the promised pension payment
This paper advocates a new approach to valuing the PBGC support by incorporating

- first, the support of the PBGC is a secondary guarantee provided to the pension funds
  - the sponsoring company provides a primal support to the pension plans
  - the sponsoring company covers the deficits of the pension fund when it does not lead to its own bankruptcy

- second, the pension fund can be prematurely terminated by an external regulator
  - premature termination is triggered by the underfunding of the pension fund;
  - premature termination means the pension fund is trusted by the PBGC
Main results

- We explicitly model the pension insurance provided by the PBGC taking account of the above two realistic aspects.
- We are able to obtain an analytical valuation formula for the insurance premium.
  - Premium highly related to the funding status of the pension fund, the investment strategies of the sponsor and the pension fund.
  - Premium increasing in the correlation between the assets of the pension fund and of the sponsoring company.
Agenda

- Introduction (√)
- Model setup
- Illustrative examples
- Conclusion
Defined benefit plan

- The pension plan is issued at time $t_0 = 0$ to a representative beneficiary.

- The benefits are paid out as a **lump-sum payment** at the beneficiary’s retirement date $R$: $B_R$
  
  ▶ $B_R$ is the prespecified benefit related to an employee’s salary and years of service.
  
  ▶ We assume that the pension liability $B_R$ is deterministic.
Pension fund’s asset process

- The pension fund invests in a constant equity-bond-mix
- The pension fund trades in the risk free bond $F$ and the risky asset $A$ in a self-financing way starting with initial wealth $X_0$

$$dX_t = \theta \cdot X_t \cdot \frac{dA_t}{A_t} + (1 - \theta) \cdot X_t \cdot \frac{dF_t}{F_t}$$

$$= X_t (r + \theta (\mu - r)) + \theta \sigma X_t \, dW_t^1$$

$$dA_t = \mu A_t \, dt + \sigma A_t \, dW_t^1 \quad \text{(risky asset)}$$

$$dF_t = rF_t \, dt \quad \text{(risk-free asset)}$$

- $\theta$ is the fraction of wealth invested in the risky asset $A$ and the remaining $(1 - \theta)$ fraction invested in the risk free bond $F$
- $W^1$ is a standard Brownian motion under the market probability measure $P$
Sponsoring company’s asset process

- The sponsoring company’s assets also follow Black-Scholes dynamics with instantaneous rate of return $\mu_c > 0$ and volatility $\sigma_c > 0$

$$dC_t = \mu_c C_t dt + \sigma_c C_t (\rho dW^1_t + \sqrt{1 - \rho^2} dW^2_t)$$

- $W^2$ is again a standard Brownian motion under the market measure $P$, independent of $W^1$.
- The sponsoring corporation’s and the pension fund’s assets are correlated with a correlation coefficient $\rho \in [-1, 1]$. 

Regulatory monitoring

- Assumption: an external pension regulator continuously monitors the pension fund’s asset.
- The regulator closes the pension fund and gives it to the PBGC if at any $t \leq R$:

$$X_t \leq \eta B_R e^{-r(R-t)}$$

- The regulatory threshold: $\eta B_R e^{-r(R-t)}$ for time $t \leq R$. We assume $\eta \in (0, 1)$.
- Let $\tau = \inf \{t | X_t \leq \eta B_R e^{-r(R-t)} \}$
- If the pension fund’s assets over-perform the threshold all the time, the pension fund is terminated naturally at time $R$. 
Payoff structure of the sponsor support

- The sponsor support depends on its own funding situation:
  - Default of the sponsoring company as the event that its assets are insufficient to pay back its own outstanding debt $\phi C_0 e^{gt}$
  - Providing the support shall not lead to its own default
    - when the sponsoring company is already defaulted
      $\Rightarrow$ no support
    - the sponsor’s assets perform extremely well
      $\Rightarrow$ full support
    - the sponsor’s assets perform moderately
      $\Rightarrow$ partial support

- Sponsor guarantees: $\Phi_c(\tau)$ (the payment sponsor makes at $\tau$); $\Phi_c(R)$ (the payment sponsor makes at time $R$)
Sponsor support given premature termination $\Phi_{C}(\tau)$

- Deficit of the pension fund at $\tau$: $B_{R}e^{-(R-\tau)} - X_{\tau}$

- The funding situation of the sponsoring company is examined at $\tau$:
  
  a) Its assets are even insufficient to pay back its own outstanding debt $C_{\tau} < \phi C_{0}e^{g\tau} \Rightarrow$ no support
  
  b) The company’s assets perform extremely well, when $C_{\tau} \geq \phi C_{0}e^{g\tau} + B_{R}e^{-r(R-\tau)} - X_{\tau} \Rightarrow$ full support $B_{R}e^{-r(R-\tau)} - X_{\tau}$
  
  c) The sponsoring company’s assets perform moderately, when $\phi C_{0}e^{g\tau} < C_{\tau} < \phi C_{0}e^{g\tau} + B_{R}e^{-r(R-\tau)} - X_{\tau} \Rightarrow$ partial support $C_{\tau} - \phi C_{0}e^{g\tau}$
Sponsor support at maturity date $R$ ($\Phi_c(R)$)

- Deficit of the pension fund at $R$: $$(B_R - X_R)1_{\{X_R < B_R\}}$$

$$\Phi_c(R) = \begin{cases} 
0 & C_R < C_0e^{gR} \\
B_R - X_R & C_R > \phi C_0e^{gR} + (B_R - X_R) \text{ and } X_R < B_R \\
C_R - \phi C_0e^{gR} & \phi C_0e^{gR} < C_R < \phi C_0e^{gR} + (B_R - X_R) \text{ and } X_R < B_R 
\end{cases}$$

- We can express the entire support provided by the sponsoring company as:

$$\Phi_c = \Phi_c(\tau)1_{\{\tau \leq R\}} + \Phi_c(R)1_{\{\tau > R\}}.$$
Payoff profile of the PBGC insurance

- We assume here that the PBGC covers all the residual deficits that the sponsoring company is unable to cover:
  - Upon termination by the regulator, the insurance of the PBGC is
    \[ G(\tau) = (B_R e^{-r(R-\tau)} - X_\tau) - \Phi_c(\tau) \]
  - Upon natural termination, the insurance of the PBGC is
    \[ G(R) = (B_R - X_R)1_{\{X_R \leq B_R\}} - \Phi_c(R). \]
  - More compactly, the insurance of the PBGC
    \[ G = G(\tau)1_{\{\tau \leq R\}} + G(R)1_{\{\tau > R\}}. \]
The payoff profile for the PBGC insurance can be understood as exotic options ⇒ Risk-neutral pricing

- One can first adjust the probability measure such that it incorporates the effect of risk, then takes the expected discounted value under the adjusted measure.

- This new measure is the so-called risk-neutral probability measure $P^*$ and the valuation method is risk-neutral pricing.
Market cost of the PBGC insurance

- The market cost (premium) paid by the sponsoring company to the PBGC is the expected discounted insurance payoff under the risk neutral probability measure:

  \[ G_0 = E^* \left[ e^{-r R} G(R) 1_{\{\tau > R\}} \right] + E^* \left[ e^{-r \tau} G(\tau) 1_{\{\tau \leq R\}} \right] \]

  where \( E^* \) denotes the expected value under the risk-neutral measure \( P^* \).

- We can also calculate “pseudo-premium” the sponsor would have obtained as a compensation for providing support

  \[ \Phi_c(0) = E^* \left[ e^{-r R} \Phi_c(R) 1_{\{\tau > R\}} \right] + E^* \left[ e^{-r \tau} \Phi_c(\tau) 1_{\{\tau \leq R\}} \right] \]

- Closed-form solutions are obtained.
Way of solutions

Write down the assets processes under the risk-neutral measure

- **Premium (premature termination)**
  - At time $\tau$, it holds
    \[
    X_\tau = X_0 \exp \left\{ \left( r - \frac{1}{2} \sigma^2 \theta^2 \right) \tau + \sigma \theta W^{1*}_\tau \right\} = \eta B_R e^{-r(R-\tau)} \Rightarrow W^{1*}_\tau
    \]
  - Substitute $W^{1*}_\tau$ in $C_\tau$ (depending on $W^{1*}_\tau$ and $W^{2*}_\tau$)
  - Transform the condition $C_0 e^{g\tau} < C_\tau < \phi C_0 e^{g\tau} + (B_\tau - X_\tau)$ to the condition $d_2(\tau) < W^{2*}_\tau < d_1(\tau)$
  - Use the density of the first-hitting $\tau$

- **Premium (natural termination)**
  - Use the property of iterated expectations
  - We use the fact that $\ln X$ and $\ln C$ follow the cumulative bivariate normal distribution with correlation coefficient $\rho$.
  - Use the probability $P^*(\tau > R)$
Parameter choices

- For the numerical illustration, we fix the relevant parameters as follows:

\[
\begin{align*}
\theta &= 0.6, \quad \phi = 0.6, \quad r = 0.05, \quad g = 0.02, \quad \eta = 0.8, \\
\sigma &= 0.20, \quad \sigma_c = 0.3333, \quad R = 15, \quad X_0 = 100, \quad B_R = 190.3
\end{align*}
\]

- \(\theta\): weight in equity holding
- \(\phi\): leverage ratio; \(g\): growth rate of debt level
- \(\eta\): regulatory parameter
- \(\sigma, \sigma_c\): volatility of the pension fund and the company’s assets
Premium for PBGC and sponsoring company

<table>
<thead>
<tr>
<th>$\rho = -0.5$</th>
<th>$\rho = -0.25$</th>
<th>$\rho = 0$</th>
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<tbody>
<tr>
<td>Premium</td>
<td>Premium</td>
<td>Premium</td>
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<tr>
<td>Sponsor</td>
<td>Sponsor</td>
<td>Sponsor</td>
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<tr>
<td>PBGC</td>
<td>PBGC</td>
<td>PBGC</td>
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<tr>
<td>8.568</td>
<td>7.601</td>
<td>6.602</td>
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<tr>
<td>2.240</td>
<td>3.207</td>
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<tr>
<td>PBGC</td>
<td>PBGC</td>
</tr>
<tr>
<td>5.498</td>
<td>4.181</td>
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<tr>
<td>5.310</td>
<td>6.627</td>
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Implied premium for providing the sponsor guarantee and PBGC guarantee.
Risk-based premium

<table>
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<tr>
<th>$\theta$</th>
<th>$\eta = 0.8$</th>
<th>$\rho = -0.5$</th>
<th>$\rho = 0$</th>
<th>$\rho = 0.5$</th>
<th>Vanilla put</th>
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<tr>
<td>0.1</td>
<td>0.033</td>
<td>0.145</td>
<td>0.258</td>
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<td>0.294</td>
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<tr>
<td>0.3</td>
<td>1.025</td>
<td>2.438</td>
<td>3.984</td>
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<td>4.670</td>
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<tr>
<td>0.5</td>
<td>1.957</td>
<td>3.891</td>
<td>6.221</td>
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<td>10.116</td>
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<tr>
<td>0.7</td>
<td>2.433</td>
<td>4.362</td>
<td>6.760</td>
<td></td>
<td>15.673</td>
</tr>
</tbody>
</table>

Fair premium for PBGC’s insurance as a function of $\theta$ and $\rho$. 

$\eta = 0.8$. $\theta = 0$.
Premium as a function of vola of the company’s asset $\sigma_c$

Fair premium $G_0$ as a function of the volatility of the company’s assets $\sigma_c$. 
Premium as a function of the regulatory parameter $\eta$

![Diagram showing the premium as a function of the regulatory parameter $\eta$ with different values of $\rho$.

- Red line: $\rho = -0.5$
- Black line: $\rho = 0$
- Blue line: $\rho = 0.5$]
Summary

- We model the pension insurance provided by the PBGC and determine the risk-based premium for this insurance.

- The premium is highly related to the funding status of the pension fund, the investment strategies of the sponsor and the pension fund, and particularly the correlation between these two portfolios.

- Vanilla put can be considered as a limiting case and provides a price upper bound to our case.
Premium responsible for premature default

\[
E^* \left[ e^{-r\tau} G(\tau) 1_{\{\tau \leq R\}} \right] \\
= \int_0^T e^{-rs} (1 - \eta) B_R e^{-r(R-s)} N \left( \frac{d_2(s)}{\sqrt{s}} \right) f(s) ds \\
+ \int_0^T e^{-rs} \left( (1 - \eta) B_R e^{-r(R-s)} + \phi C_0 e^{gs} \right) \left( N \left( \frac{d_1(s)}{\sqrt{s}} \right) - N \left( \frac{d_2(s)}{\sqrt{s}} \right) \right) f(s) ds \\
- \int_0^T e^{-rs} C_0 \exp \left\{ \left( r - \frac{1}{2} \sigma_c^2 \right) s + \sigma_c \rho \ln \frac{\eta B_R e^{-r(R-s)}}{X_0} - (r - \frac{1}{2} \theta^2 \sigma^2) s \right\} \\
\cdot \left( \int \frac{d_1(s)}{\sqrt{s}} \exp \{ \sigma_c \sqrt{1 - \rho^2} \sqrt{s}x \} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \right) f(s) ds
\]

where \( f(s) \) is the density of the first hitting time \( \tau \) under \( P^* \).
Premium responsible for maturity termination

\[ E^* \left[ e^{-rR} G(R) 1_{\{\tau > R\}} \right] \]

\[ = P^* (\tau > R) \int_{d_{x1}}^{d_{x2}} \int_{d_{y1}}^{d_{y2}} e^{-rR} \left( B_R - X_0 \exp \left\{ \left( r - \frac{1}{2} \theta^2 \sigma^2 \right) R + \theta \sigma \sqrt{R} x \right\} \right) \]

\[ \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp \left\{ -\frac{1}{2(1 - \rho^2)} (x^2 + y^2 - 2\rho xy) \right\} dydx \]

\[ + P^* (\tau > R) \int_{d_{x1}}^{d_{x2}} \int_{d_{y1}}^{d_{y2}} e^{-rR} \left( \left( B_R - X_0 \exp \left\{ \left( r - \frac{1}{2} \theta^2 \sigma^2 \right) R + \theta \sigma \sqrt{R} x \right\} \right) \]

\[ - \left( C_0 \exp\{ (r - \frac{1}{2} \sigma_c^2) R + \sigma_c \sqrt{R} y \} - \phi C_0 e^{\gamma R} \right) \)

\[ \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp \left\{ -\frac{1}{2(1 - \rho^2)} (x^2 + y^2 - 2\rho xy) \right\} dydx \]