Time-Consistent and Market-Consistent Actuarial Valuations

Antoon Pelsser\textsuperscript{1}

\textsuperscript{1}Maastricht University & Netspar
Email: a.pelsser@maastrichtuniversity.nl

30 April 2010
DGVFM Scientific Day – Bremen
Standard actuarial premium principles usually consider “static” premium calculation:
- What is price today of insurance contract with payoff at time $T$?

Actuarial premium principles typically “ignore” financial markets

Financial pricing considers “dynamic” pricing problem:
- How does price evolve over time until time $T$?

Financial pricing typically “ignores” unhedgeable risks

Examples:
- Pricing very long-dated cash flows $T \sim 30 – 100$ years
- Pricing long-dated options $T > 5$ years
- Pricing pension & insurance liabilities
- Pricing employee stock-options
In this paper I want to combine
1. Time-Consistent pricing operators, see [Jobert and Rogers, 2008]
2. Market-Consistent pricing operators, see [Malamud et al., 2008]

Both references concentrate on discrete-time algorithms

I will be interested in continuous-time limits of these discrete algorithms for different actuarial premium principles:
1. Variance Principle
2. Mean Value Principle
3. Standard-Deviation Principle
Content of This Talk

1 Pure Insurance Risk
   - Diffusion Model for Insurance Risk
   - Variance Principle ($\rightarrow$ exponential indiff. pricing)
   - Standard-Dev. Principle ($\rightarrow \mathbb{E}[]$ under new measure)
   - Cost-of-Capital Principle ($\rightarrow$ St.Dev price)
   - Davis Price, see [Davis, 1997]
     - St.Dev is “small perturbation” of Variance price

2 Financial & Insurance Risk
   - Diffusion Model for Financial Risk
   - Market-Consistent Pricing
   - Variance Principle
   - Numerical Illustration

3 Conclusions
Consider unhedgeable insurance process $y$:

$$dy = a(t, y) \, dt + b(t, y) \, dW$$

To keep math simple, concentrate on diffusion setting.

Discretisation scheme as binomial tree:

$$y(t + \Delta t) = y(t) + a\Delta t + \left\{ \begin{array}{ll}
+b\sqrt{\Delta t} & \text{with prob. } \frac{1}{2} \\
-b\sqrt{\Delta t} & \text{with prob. } \frac{1}{2}
\end{array} \right.$$
Time Consistency

- Time Consistent price $\pi(t, y)$ satisfies property

$$\pi[f(y(T))|t, y] = \pi[\pi[f(y(T))|s, y(s)]|t, y] \quad \forall t < s < T$$

- Price of today of holding claim until $T$ is the same as buying claim half-way at time $s$ for price $\pi(s, y(s))$

- “Semi-group property”

- Similar idea as “tower property” of conditional expectation
Variance Principle

- Actuarial Variance Principle $\Pi^v$:

$$\Pi_t^v[f(y(T))] = \mathbb{E}_t[f(y(T))] + \frac{1}{2} \alpha \text{Var}_t[f(y(T))]$$

- $\alpha$ is Absolute Risk Aversion
- Apply $\Pi^v$ to one binomial time-step to obtain price $\pi^v$:

$$\pi^v(t, y(t)) = \mathbb{E}_t[\pi^v(t + \Delta t, y(t + \Delta t))] + \frac{1}{2} \alpha \text{Var}_t[\pi^v(t + \Delta t, y(t + \Delta t))]$$

- Note: we omit discounting for now
Assume $\pi^v(t, y)$ admits Taylor approximation in $y$

Evaluate Var.Princ. for binomial step & take limit for $\Delta t \to 0$

- Same as derivation of Feynman-Kac, but for $E[] + \frac{1}{2} \alpha \text{Var}[]$

This leads to pde for $\pi^v$:

$$\pi_t^v + a \pi_y^v + \frac{1}{2} b^2 \pi_{yy}^v + \frac{1}{2} \alpha (b \pi_y^v)^2 = 0$$

Note, non-linear term = “local unhedgeable variance” $b^2 (\pi_y^v)^2$

Find general solution to this non-linear pde via log-transform:

$$\pi^v(t, y) = \frac{1}{\alpha} \ln E_t \left[ e^{\alpha f(y(T))} \bigg| y(t) = y \right].$$

Exponential indifference price, see [Henderson, 2002] or [Musiela and Zariphopoulou, 2004]
Include Discounting

- We should include discounting into our pricing
- Absolute Risk Aversion $\alpha$ is not “unit-free”, but has unit $1/\€$
  - This conveniently compensates the unit $(\€)^2$ of $\text{Var}[]$...
- Therefore, “$\alpha$-today” is different than “$\alpha$-tomorrow”
- Relative Risk Aversion $\gamma$ is unit-free
- Express ARA relative to “benchmark wealth” $X_0 e^{rT}$
- Explicit notation: $\alpha \rightarrow \gamma/X_0 e^{rT}$ leads to pde:

$$\begin{align*}
\pi_t^v + a\pi_y^v + \frac{1}{2} b^2 \pi_{yy}^v + \frac{1}{2} \frac{\gamma}{X_0 e^{rT}} (b\pi_y^v)^2 - r\pi^v &= 0 \\
\pi^v(t, y) &= \frac{X_0 e^{rt}}{\gamma} \ln \mathbb{E} \left[ e^{\frac{\gamma}{X_0 e^{rT}} f(y(T))} \bigg| y(t) = y \right]
\end{align*}$$

- Note: express all prices in discounted terms
Backward Stochastic Differential Equations

- Pricing PDE:
  \[ \pi_t^v + a\pi_y^v + \frac{1}{2} b^2 \pi_{yy}^v + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} (b\pi_y^v)^2 - r\pi^v = 0 \]

- This non-linear PDE, represents the solution to a so-called BSDE for the triplet of processes \((y_t, Y_t, Z_t)\)
  \[
  \begin{cases}
  dy_t = a(t, y_t) \, dt + b(t, y_t) \, dW_t \\
  dY_t = -g(t, y_t, Y_t, Z_t) \, dt + Z_t \, dW_t \\
  Y_T = f(y(T))
  \end{cases}
  \]

- with “generator” \(g(t, y, Y, Z) = \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} Z^2 - rY\).

- Recent literature studies uniqueness & existence of solutions to BSDE’s, see [El Karoui et al., 1997]

- Via BSDE’s we can study time-consistent pricing operators in a much more general stochastic setting. But we will not pursue this here.
Mean Value Principle

- Generalise to *Mean Value Principle*

\[ \Pi_t^m[f(y(T))] = \nu^{-1}(\mathbb{E}_t[\nu(f(y(T)))] \]

- for any function \( \nu() \) which is a convex and increasing
  - Exponential pricing is special case with \( \nu(x) = e^{\alpha x} \)

- Do Taylor-expansion & limit \( \Delta t \to 0 \):

\[
\pi_t^{mf} + a\pi_y^{mf} + \frac{1}{2} b^2 \pi_y^{mf} + \frac{1}{2} \frac{\nu''(\pi^{mf})}{\nu'(\pi^{mf})} (b\pi_y^{mf})^2 = 0
\]

- Note: \( \pi^{mf}(t, y) := \pi^m(t, y)/e^{rt} \) is price expressed in discounted terms

- Interpretation as generalised Variance Principle with “local risk aversion” term: \( \nu''()/\nu'() \)
Actuarial Standard-Deviation Principle:

\[ \Pi^s_t[f(y(T))] = \mathbb{E}_t[f(y(T))] + \beta \sqrt{\text{Var}_t[f(y(T))]} \]

Pay attention to “time-scales”:
- Expectation scales with \( \Delta t \)
- St. Dev. scales with \( \sqrt{\Delta t} \)

Thus, we should take \( \beta \sqrt{\Delta t} \) to get well-defined limit
- Note: \( \beta \) has unit \( 1/\sqrt{\text{time}} \)
Pricing PDE

- Do Taylor-expansion & limit $\Delta t \to 0$:
  \[
  \pi_t^s + a\pi_y^s + \frac{1}{2} b^2 \pi_{yy}^s + \beta \sqrt{(b\pi_y^s)^2 - r\pi^s} = 0
  \]

- Again, non-linear pde. But if $\pi^s$ is monotone in $y$ then
  \[
  \pi_t^s + (a \pm \beta b)\pi_y^s + \frac{1}{2} b^2 \pi_{yy}^s - r\pi^s = 0
  \]

- “Upwind” drift-adjustment into direction of risk
  \[
  \pi^s(t, y) = E_t^S [f(y(T))|y(t) = y]
  \]
Cost-of-Capital Principle

- Cost-of-Capital principle, popular by practitioners
  - Used in QIS4-study conducted by CEIOPS
- Idea: hold buffer-capital against unhedgeable risks. Borrow from shareholders by giving “excess return” $\delta$
- Define buffer via Value-at-Risk measure:

$$\Pi_t^c[f(y(T))] = \mathbb{E}_t[f(y(T))] + \delta \text{VaR}_{q,t} \left[ f(y(T)) - \mathbb{E}_t[f(y(T))] \right].$$
Again, pay attention to “time-scaling”:
- First, scale VaR back to per annum basis with $1/\sqrt{\Delta t}$
- Then, $\delta$ is like interest rate, so multiply with $\Delta t$
- Net scaling: $\delta \Delta t/\sqrt{\Delta t} = \delta \sqrt{\Delta t}$.

Limit: for small $\Delta t$ the VaR behaves as $\Phi^{-1}(q) \times \text{St.Dev}$. Hence, limiting pde is same as $\pi^s$ but with $\beta = \Phi^{-1}(q)\delta$.

Conclusion: In the limit for $\Delta t \rightarrow 0$, CoC pricing is the same as st.dev. pricing
The variance price $\pi^v$ is “hard” to calculate, the st.dev. price $\pi^s$ is “easy” to calculate.

Can we make a connection between these two concepts?

“Yes, we can!” using small perturbation expansion.

Consider existing insurance portfolio with price $\pi^v(t, y)$, now add “small” position with price $\varepsilon \pi^D(t, y)$. Subst. into pde:

$$(\pi^v_t + \varepsilon \pi^D_t) + a(\pi^v_y + \varepsilon \pi^D_y) + \frac{1}{2} b^2 (\pi^v_{yy} + \varepsilon \pi^D_{yy}) + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} b^2 \left( (\pi^v_y)^2 + 2\varepsilon \pi^v_y \pi^D_y + \varepsilon^2 (\pi^D_y)^2 \right) - r(\pi^v + \varepsilon \pi^D) = 0$$

$\pi^v()$ solves the pde, cancel $\pi^v$-terms.
Pricing PDE

- Simplify pde, and divide by $\varepsilon$:

$$
\pi_t^D + a\pi_y^D + \frac{1}{2} b^2 \pi_{yy}^D + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} b^2 \left( 2\pi^v_y \pi^D_y + \varepsilon (\pi^D_y)^2 \right) - r\pi^D = 0
$$

- Approximation: ignore “small” $\varepsilon$-term

$$
\pi_t^D + \left( a + \frac{\gamma}{X_0 e^{rt}} b^2 \pi^v_y \right) \pi_y^D + \frac{1}{2} b^2 \pi_{yy}^D + r\pi^D = 0
$$

$$
\pi^D(t, y) = \mathbb{E}^D_t [f(y(T)) | y(t) = y]
$$

- Davis price $\pi^D$ is defined only “relative” to existing price $\pi^v$ of insurance portfolio

- Note, drift-adjustment of st.dev. price scales with $b$
Investigate environment with financial risk that can be traded (and hedged!) in financial market and non-traded insurance risk.

Model financial risk as [Black and Scholes, 1973] economy. Model return process $x_t = \ln S_t$ under real-world measure $\mathbb{P}$:

$$dx = \left( \mu(t, x) - \frac{1}{2} \sigma^2(t, x) \right) dt + \sigma(t, x) dW_f$$

Binomial time-step:

$$x(t + \Delta t) = x(t) + \left( \mu - \frac{1}{2} \sigma^2 \right) \Delta t + \begin{cases} +\sigma \sqrt{\Delta t} & \text{with } \mathbb{P}\text{-prob. } \frac{1}{2} \\ -\sigma \sqrt{\Delta t} & \text{with } \mathbb{P}\text{-prob. } \frac{1}{2} \end{cases}$$
No-arbitrage pricing

- BS-economy is arbitrage-free and complete ⇔ unique martingale measure $\mathbb{Q}$.
- No-arbitrage pricing operator for financial derivative $F(x(T))$:
  \[
  \pi^Q(t, x) = e^{-r(T-t)}E^Q_t[F(x(T))]
  \]
- Binomial step for $x$ under measure $\mathbb{Q}$:
  \[
  x(t + \Delta t) = x(t) + (\mu - \frac{1}{2}\sigma^2)\Delta t + \begin{cases} 
  +\sigma\sqrt{\Delta t} & \text{with } \mathbb{Q}\text{-prob. } \frac{1}{2} \left(1 - \frac{\mu-r}{\sigma} \sqrt{\Delta t}\right) \\
  -\sigma\sqrt{\Delta t} & \text{with } \mathbb{Q}\text{-prob. } \frac{1}{2} \left(1 + \frac{\mu-r}{\sigma} \sqrt{\Delta t}\right)
  \end{cases}
  \]
- Quantity $(\mu - r)/\sigma$ is Radon-Nikodym exponent of $d\mathbb{Q}/d\mathbb{P}$
- Quantity $(\mu - r)/\sigma$ is also known as market-price of financial risk.
Joint discretisation for processes $x$ and $y$ using “quadrinomial” tree with correlation $\rho$ under measure $\mathbb{P}$:

<table>
<thead>
<tr>
<th>State:</th>
<th>$y + \Delta y$</th>
<th>$y - \Delta y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + \Delta x$</td>
<td>$\left( \frac{1+\rho}{4} \right)$</td>
<td>$\left( \frac{1-\rho}{4} \right)$</td>
</tr>
<tr>
<td>$x - \Delta x$</td>
<td>$\left( \frac{1-\rho}{4} \right)$</td>
<td>$\left( \frac{1+\rho}{4} \right)$</td>
</tr>
</tbody>
</table>

Positive correlation increases probability of joint “++” or “−−” co-movement
Market-Consistent Pricing

- We are looking for *market-consistent* pricing operators, see e.g. [Malamud et al., 2008]

**Definition**

A *pricing operator* \( \pi() \) is *market-consistent* if for any financial derivative \( F(x(T)) \) and any other claim \( G(t, x, y) \) we have

\[
\pi_{F+G}(t, x, y) = e^{-r(T-t)} \mathbb{E}_t^Q [F(x(T))] + \pi_G(t, x, y).
\]

- Observation: generalised notion of “translation invariance” for all financial risks
Intuition: construct MC pricing in two steps using conditional expectations, see also: [Carmona, 2008], Chap. 1

First: condition on financial risk & use actuarial pricing for “pure insurance” risk

\[
\pi^v(t + \Delta t | x \pm) := \mathbb{E}[\pi^v(t + \Delta t | x \pm)] + \frac{1}{2} \gamma X_0 e^{r(t+\Delta t)} \mathrm{Var}[\pi^v(t + \Delta t | x \pm)]
\]

\[
\mathbb{E}[\pi^v(t + \Delta t | x +)] = \left(\frac{1 + \rho}{2}\right) \pi^v_{++} + \left(\frac{1 - \rho}{2}\right) \pi^v_{+-}
\]

\[
\mathrm{Var}[\pi^v(t + \Delta t | x +)] = \left(\frac{1 - \rho^2}{4}\right) \left(\pi^v_{++} - \pi^v_{+-}\right)^2
\]

\[
\mathbb{E}[\pi^v(t + \Delta t | x -)] = \left(\frac{1 + \rho}{2}\right) \pi^v_{-+} + \left(\frac{1 - \rho}{2}\right) \pi^v_{--}
\]

\[
\mathrm{Var}[\pi^v(t + \Delta t | x -)] = \left(\frac{1 - \rho^2}{4}\right) \left(\pi^v_{-+} - \pi^v_{--}\right)^2.
\]

For \(\rho = 1\) or \(\rho = -1\) no unhedgeable risk left \(\Rightarrow \mathrm{Var} = 0\)
Pricing PDE

- Second: use no-arbitrage pricing for “artificial” financial derivative

\[
\pi^v(t, x, y) = e^{-r\Delta t} E^Q[\pi^v(t + \Delta t | x \pm)]
\]

\[
= e^{-r\Delta t} \left( \frac{1}{2} \left( 1 - \frac{\mu - r}{\sigma} \sqrt{\Delta t} \right) \pi^v(t + \Delta t | x+) + \frac{1}{2} \left( 1 + \frac{\mu - r}{\sigma} \sqrt{\Delta t} \right) \pi^v(t + \Delta t | x-) \right)
\]

- Do Taylor-expansion & limit $\Delta t \to 0$:

\[
\pi^v_t + (r - \frac{1}{2} \sigma^2) \pi^v_x + (a - \rho b \frac{\mu - r}{\sigma}) \pi^v_y + \frac{1}{2} \sigma^2 \pi^v_{xx} + \rho \sigma b \pi^v_{xy} + \frac{1}{2} b^2 \pi^v_{yy} + \frac{\gamma}{X_0 e^{rt}} (1 - \rho^2)(b \pi^v_y)^2 - r \pi^v = 0
\]

- Impact on $x$: “$Q$-drift” \((r - \frac{1}{2} \sigma^2)\)
- Impact on $y$: adjusted drift \((a - \rho b \frac{\mu - r}{\sigma})\)
- Non-linear term for “locally unhedgeable variance” \((1 - \rho^2)(b \pi^v_y)^2\)
Pure Insurance Payoff

- Unfortunately, we cannot solve the non-linear pde in general

- Special case: consider pure insurance payoff (and constant MPR \( \frac{\mu - r}{\sigma} \)), then no (explicit) dependence on \( x \)

\[
\pi_t^v + (a - \rho b\frac{\mu - r}{\sigma}) \pi_y^v + \frac{1}{2} b^2 \pi_{yy}^v + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} (1 - \rho^2) (b \pi_y^v)^2 - r \pi^v = 0
\]

- Note that for \( \rho \neq 0 \) the pde is different from the “pure insurance” pde

- Via correlation with financial market we can still hedge part of the insurance risk

- Note: incomplete market \( \Rightarrow \) no martingale representation, therefore delta-hedge is not \( \pi_x^v \)

  - In fact: hold \( \rho b \pi_y^v / \sigma \) in \( x \) as hedge

  - Economic explanation for drift-adjustment in \( y \), a kind of “quanto-adjustment”
Pure Insurance Payoff (2)

- General solution via log-transform:

  \[ \pi^\nu(t, y) = \frac{X_0 e^{rt}}{\gamma(1 - \rho^2)} \ln \mathbb{E}^{\tilde{P}} \left[ e^{\frac{\gamma(1-\rho^2)}{X_0 e^{rT}} f(y(T))} \bigg| y(t) = y \right] \]

- Measure \( \tilde{P} \) induces drift-adjusted process for \( y \)

- See also, [Henderson, 2002] and [Musiela and Zariphopoulou, 2004] who derived this solution in the context of exponential indifference pricing

- We can generalise to Mean Value Principle for any convex function \( \nu() \)
Numerical Illustration

- Consider “unit-linked” insurance contract with payoff:
  \[ y(T)S(T) = y(T)e^{x(T)} \]
- Numerical calculation in quadrinomial tree with 5 time-steps of 1 year
- “Naive” hedge is to hold \( y(t) \) units of share \( S(t) \)
- In fact: hold \( \pi_y^V + \rho b \pi_y^V / \sigma \) as hedge
- MC Variance hedge also builds additional reserve as “buffer” against unhedgeable risk
Numerical Illustration (2)

- Investigate impact of correlation $\rho$
- Compare $\rho = 0.50$ (left) and $\rho = 0$ (right)

Positive correlation leads to higher delta, as this also hedges part of insurance risk: hold $\pi_X^V + \rho b\pi_Y^V/\sigma$ as hedge

- Price for $\rho = 0.00$ at $t = 0$ is €26.75
- Price for $\rho = 0.50$ at $t = 0$ is €18.79, due to less unhedgeable risk
- Price for $\rho = 0.99$ at $t = 0$ is €1.98, due to drift-adjustment
MC Standard-Deviation Pricing

- Again, do two-step construction
- First: condition on financial risk & use actuarial pricing for “pure insurance” risk
  
  \[
  \pi^s(t + \Delta t|x\pm) := \mathbb{E}[\pi^s(t + \Delta t)|x\pm] + \delta \sqrt{\Delta t} \sqrt{\text{Var}[\pi^s(t + \Delta t)|x\pm]}
  \]

- Second: do no-arbitrage valuation under \( Q \)
- This leads to linear pricing pde (if \( \pi^s(t, x, y) \) monotone in \( y \)):

  \[
  \pi_t^s + (r - \frac{1}{2} \sigma^2) \pi_x^s + \left( a - \rho b \frac{\mu - r}{\sigma} \pm \delta \sqrt{1 - \rho^2} b \right) \pi_y^s + \frac{1}{2} \sigma^2 \pi_{xx}^s + \rho \sigma b \pi_{xy}^s + \frac{1}{2} b^2 \pi_{yy}^s - r \pi^s = 0
  \]

- Drift adjustment for \( y \) is now combination of “hedge cost” plus “upwind” risk-adjustment \( \pm \delta \sqrt{1 - \rho^2} b \)
We can again consider Davis price, by “small perturbation” expansion

This leads to pricing pde:

\[
\pi_t^D + (r - \frac{1}{2}\sigma^2)\pi_x^D + \left(a - \rho b \frac{\mu - r}{\sigma} + \frac{\gamma}{X_0 e^{rt}} (1 - \rho^2) b^2 \pi_y^v\right) \pi_y^D + \frac{1}{2} \sigma^2 \pi_{xx}^D + \rho \sigma b \pi_{xy}^D + \frac{1}{2} b^2 \pi_{yy}^D - r \pi^D = 0
\]

Drift adjustment for \(y\) is now combination of “hedge cost” plus risk-adjustment \(\frac{\gamma}{X_0 e^{rt}} (1 - \rho^2) b^2 \pi_y^v\)

Davis price defined relative to existing price \(\pi^v(t, x, y)\)

Note, st.dev. pricing depends on \(\sqrt{(1 - \rho^2)} b\)
Multi-dimensional MC Variance Price

- Vectors $x$ of asset returns ($n$-vector), $y$ of insurance risks ($m$-vector)
  
  $$dx = \mu \, dt + \Sigma^{1/2} \cdot dW_f$$
  
  $$dy = a \, dt + B^{1/2} \cdot dW$$

- Partitioned covariance matrix $C$ ($n + m$) $\times$ ($n + m$)
  
  - $P$ is $n \times m$ matrix of financial & insurance covariances
    
    $$C = \begin{pmatrix} \Sigma & P \\ P' & B \end{pmatrix}$$

- The market-price of financial risks is an $n$-vector $\Sigma^{-1} (\mu - r)$

- Multi-dim pricing pde for $\pi^y$:
  
  $$\pi^y_t + r' \pi^y_x + \left(a - P' \Sigma^{-1} (\mu - r)\right)' \pi^y_y + \frac{1}{2} \left(C_{ij} \pi^y_{ij}\right) + \frac{1}{2} \frac{\gamma}{X_0 e^{rt}} \left(\pi^y_{yy} (B - P' \Sigma^{-1} P) \pi^y_y\right) - r \pi^y = 0$$

- Note: $(B - P' \Sigma^{-1} P)$ is conditional covariance matrix of $y|x$
Multi-dimensional MC StDev Price

- Multi-dim pricing pde for $\pi^s$:

$$
\pi_t^s + r'\pi_x^s + \left( a - P'\Sigma^{-1}(\mu - r) \right)'\pi_y^s + \frac{1}{2} \left( C_{ij}\pi_{ij}^s \right) + \frac{1}{2} \delta \sqrt{\pi_{y}'(B - P'\Sigma^{-1}P)\pi_y^s - r\pi^s} = 0
$$

- Note: unlike 1-dim case, does not simplify to linear pde
- Simplification only possible if $(B - P'\Sigma^{-1}P)$ has rank 1 and all $\pi^s_i$ have same sign
Conclusions

1. Pure Insurance Risk
   - Variance Principle (\(\rightarrow\) exponential indiff. pricing)
   - Standard-Dev. Principle (\(\rightarrow\) \(E[]\) under new measure)
   - Cost-of-Capital Principle (\(\rightarrow\) St.Dev price)
   - Davis Price: St.Dev. price is “small perturbation” of Variance price

2. Financial & Insurance Risk
   - Market-Consistent Pricing: via two-step conditional expectations
   - MC Variance Principle
   - Numerical Illustration for “unit-linked” contract
References


