Rationale for a Lower Bound in Interest Rate Models

Capital Market Models: Legal Requirements

According to Article 22, Section 3 of the Commission Delegated Regulation (EU) 2015/35, a model providing projections of future financial market parameters for the valuation of technical provisions particularly has to comply with the following requirements:

a. *it generates asset prices that are consistent with asset prices observed in financial markets;*

b. *it assumes no arbitrage opportunity;*

c. *the calibration of the parameters and scenarios is consistent with the relevant risk-free interest rate term structure used to calculate the best estimate as referred to in Article 77(2) of Directive 2009/138/EC.*

Capital Market Models: Modelling Interest Rates (1)

In order to comply with the conditions a. to c. determined by Article 22, Section 3 of the Delegated Regulation (EU) 2015/35 it is not necessary to apply models that allow for arbitrarily negative interest rates.

Moreover :

- Taking into account negative rates requires the identification of future decisions and very sensitive liability reactions to be apprehended. This adds uncertainty to the Solvency 2 metrics and hinders the analysis of the results.
- The situation leading to low and sustainable negative rates is an unprecedented economic situation. Political decisions are likely to be made in this situation. These aspects can not be taken into account in the calculation of the pillar 1 metrics. This distorts the results.

Capital Market Models: Modelling Interest Rates (2)

Within the current legal framework it is appropriate and reasonable to use valuation models featuring a lower bound for interest rates.

In the following, we argue how a natural lower bound can be derived through the existence of cash.

Classification of the asset "Cash"

Bank deposits can be replaced by cash, since

- cash is a legal tender.
- cash withdrawal is not limited.
- cash storage is not limited.

- The level of demand for cash determines the volume of cash that circulates: According to the current legal situation, the volume of cash has to be increased in case of a rising demand.∗
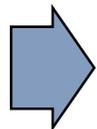
Consequently, negative interest rates on bank deposits can, ceteris paribus, be avoided through holding cash.

∗ It cannot be ruled out that future legal amendments might change this situation. However, it is necessary and reasonable to base the following considerations on the current legal framework.

Disadvantages of the asset "Cash" (1)

- Physical delivery impairs "fast transactions".
- Transaction costs increase with rising transaction volume and speed.
- Additionally, friction costs for storage, protection, insurance, etc. need to be considered : the physical possession of cash creates difficulties in terms of liquidity management and does not enable the insured to be settled quickly.

The costs of holding cash depend on the frequency, the speed and the volume of transactions.

Disadvantages of the asset "Cash" (2)

- Insurance companies are not dependent on fast transactions; especially life insurers are able to set up reliable long-term cash flow forecasts.
- Through their risk management and ALM processes, insurance companies have to maintain the necessary liquidity level to respect its commitments in any situation.
- Under these constraints, insurance companies could have the possibility to convert a significant part of their assets into cash.
- Insurers' balance of payments in relation to the investment volume is generally well-adjusted, i.e. the share of the investment volume which is subject to transactions within regular business operations is small: transactions have both relatively low frequencies and volumes.
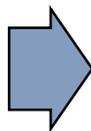
  Insurers' costs of holding cash are lower than for other sectors where fast and large transactions are more frequent.

Cost of Holding Cash at Insurance Companies

➤ **The actual level of cost from holding cash is company-specific and needs to be determined individually,**

for example[1]:

a) costs for storage and insurance:	20 bps
b) costs of transportation:	20 bps
	40 bps

➤ **Whenever the short-term interest rate falls below the costs of holding cash, the insurance company would progressively move into cash.**

- The interest rate for short-term investments (e.g. the yield on short-term government bonds) can fall below the costs of holding cash[2], since those assets can generate value to non-insurance investors due to their specific properties (e.g. fast and low-cost tradability – properties which cash does not possess).

[1] Fictitious values that do not refer to a specific company, but might be close to reality.

[2] This does not imply arbitrage opportunities for insurance companies since they cannot issue government bonds.

Assumptions for Modelling "Cash"

In the light of this reasoning, it is appropriate to introduce the asset class "cash" in valuation models.

- Prudent assumptions for the modelling of the new asset class are:

  - ✓ Holding cash provides an alternative bank deposit.

  - ✓ The cost of holding cash is fixed and independent from short-term interest rates.

  - ✓ Cash is available in unlimited quantities.

- Management rules ensure that the company moves into cash whenever yields on fixed-income investments fall below the costs of holding cash.

Explicit Modelling of the Asset Class "Cash" (1)

The valuation model becomes complex for the following reasons:

- The fixation of the nominal value of cash violates the no-arbitrage principle which implies that goods with identical future cash flows must have identical current prices ("law of one price").

- The current value of a cash available in the future is given by the nominal value minus its discounted costs (storage, protection, etc.). Accordingly, the valuation of cash and bank deposits will generally differ.

- Hence, the martingale test will fail for the asset class "cash" ("1<1").

- In order to capture the theoretical arbitrage opportunity, an insurance company would have to earn the short term interest and simultaneously lend out money at cash conditions.

- But: Only the ECB is able to issue cash and to benefit from favorable market conditions regarding cash.

Explicit Modelling of the Asset Class "Cash" (2)

- Consequently, only positive yields would be permissible for the new asset class "cash".

- A test on the absence of arbitrage opportunities in a model would have to be adjusted accordingly.

  Since the explicit modelling of cash becomes rather complex and is not yet proven in practice, we suggest an approximate approach in the following.

The impact of the „cash option" can be approximated via interest rate models with a lower bound:

Approach:

- Insurance portfolios can be valuated on the basis of interest rate models featuring a lower bound for interest rates. This reflects the current reality of insurance companies [1].

- Such a lower bound could be smaller than the cost of holding cash, in order to avoid negative impacts on stability and results of calculations.

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[1] Note that this does not necessarily imply an increase of the capital position:
Scenarios with extremely negative interest rates might not appear, but the relative weight of scenarios with low rates – that generate stress for capital positions – has to increase.

Calibration:

- The lower bound is defined taking into account all relevant and public financial data.

- Using directly costs of holding cash to calibrate a lower bound can be difficult from an operational view and such lower bound could have negative impacts on stability and results, trajectories being artificially pulled upwards.

- The notion of lower bound is rather a long-term reference that should not be modify at each reporting date (much like the UFR). A stable parameter could be defined by varying the initial conditions so as to provide a valid level in a broad spectrum of market conditions.

- The relevance of the lower bound can be verified controlling namely the replication of options and zero coupon bond prices.

Interest Rate Models with Lower Bounds

Popular models with lower bounds for interest rates:

- 1-factor Hull-White model with re-weighted paths (scenarios with interest rates below the lower bound are ignored, the remaining paths receive additional weights, which ensures that the model remains arbitrage-free)

- 1- or 2-factor Hull-White model with a transformation of the probability distribution of the modeled interest rates preserving both market-consistency and the no-arbitrage-principle (no deletion of paths)

- Shifted Libor Market Models (LMM, also: displaced LMM or LMM+)

- SABR or Constant Elasticity of Variance models

Given an appropriate determination of the lower bound's value, any of the above-listed models can be calibrated such that they fulfill the criteria for capital market models given by Article 22, Section 3 of the Delegated Regulation (EU) 2015/35 for the valuation of technical provisions.

Lower Bounds on Interest Rates in Practice

- Models with a lower bound for interest rates are widely used in Europe.

- The lower bound can be seen as an additional parameter to improve the calibration of the model.

- Furthermore, the valuation results benefit from

    - ✓ a reduced dependency of results on random seeds,

    - ✓ an improved speed of convergence as well as

    - ✓ an enhanced level of confidence due to the avoidance of potentially flawed impacts from modelled management actions in economic environments substantially different from past and current experiences

The lower bound helps to stabilize the results of the valuation.