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A Summary of a German Standard for Category 4 PRIIPs

as defined by the German Association of Actuaries (DAV)

Cologne, 15 November 2019

This paper summarises the report on findings „Ein Standardverfahren für PRIIP der Kategorie 4" which was approved by the DAV committee Life Insurance in 2017. The complete publication (in German language) is available here.

The German Association of Actuaries (Deutsche Aktuarvereinigung or DAV) is the professional body representing actuaries and Appointed Actuaries in Germany. It creates the underlying conditions enabling its members to practice their profession properly and in a technically sound manner as well as engaging in constant dialogue with all relevant national and international institutions.

Reports on findings are summaries of the results of work carried out by DAV committees or working groups,

- where their application can be freely decided upon within the framework of the code of conduct,
- that should inform discussion of the current opinion among actuaries or also among the broader public.

As working results of a single committee, they do not, for the time being, represent any recognised position within the DAV.

Contents

1. Introduction

- This paper presents a brief summary of a German standard that is used for the calculation of risk indicators and performance scenarios of German insurance-based investment products (IBIPs). The objective of this paper is twofold: first it describes the basis of a forward-looking stochastic model applied. Second, it explains in general terms how the core ideas behind this model can be directly applied to other PRIIPs and PEPP or modified in order to capture specificities of other products. We believe that this summary can contribute to the convergence of the different methods used for different products.
- The PRIIPs RTS impose different methodologies for the calculation of the market risk and performance scenarios for different products: Category 2 encompasses linear products that are wholly dependent on the market and uses the Cornish Fisher methodology. Category 3 includes structured products that are wholly dependent on the market and applies the Bootstrap methodology.
- Category 4 covers products whose values depend in part on factors not observed in the market in accordance with point 7 of Annex II of the RTS. The German IBIPs cannot in general be separated in components in a meaningful way. Therefore, they are treated as a unit. For this reason, the paper does not address the separation in components mentioned in point 27 of Annex II. For products of this category the market risk measure (MRM) is determined in accordance with point 27 of Annex II of the RTS using a well-recognised industry or regulatory standard.
- This paper describes the basis of a standard that is applied in Germany for category 4 IBIPs. The basic idea behind this standard can be seen at European level as a starting point for a forward-looking model that can be in general applied to other PRIIPs in the scope of the PRIIPs Regulation. It can also be developed further to capture the specificities of other products, as it is for example done in Austria textbooks[1], where the German model is adjusted to capture the specificities of Austrian products. Furthermore, this methodology can be applied as a basis for the currently developed PEPP due to its long-term stability.
- The standard described in this paper has been developed by the German Association of Actuaries (DAV) ("DAV standard" in the following). It was welcomed by the German supervisor BaFin. This standard uses acknowledged standard capital market models, particularly relying on well-known and approved equity and interest rate models that can be found in standard

[1] For information on the Austrian standard see https://avoe.at/wp-content/uploads/2019/03/Leitfaden-AV%C3%96-Branchenstandard-PRIIP-Kategorie-4-02_2019-final.pdf

financial mathematics textbooks[2]: It is based on the already existing standard for subsidised pension products (so called "Riester" and "Basis-Rente") in Germany that was originated by the *Produktinformationsstelle Altersvorsorge*[3] for classifying subsidised pension products (see PIA (2017)[4]) (hereinafter "PIA model"). Furthermore the industries simulation model for Solvency II by the German insurance association (GDV) uses the same or similar capital market models (see DAV (2017)). In the German insurance industry, stochastic modelling of insurance-based investment products has been done for some time using similar methods by, for example, market participants such as ifa Ulm using ifa-SARA and Morgen & Morgen using the Volatium model[5]. In thematically related academic works (e.g. Graf et al. (2012) and *Aktuar Aktuell* (2009)) similar approaches for stochastic modelling of insurance-based investment products have been used. Within this context Graf and Korn (2019) showed recently the advantages of a forward-looking simulation-based approach for the assessment of risk and return.

- The DAV standard uses the capital market calibration provided by PIA. It is partially based on long-term averages and could be calibrated in a way to generate stable results for PRIIPs. If the idea behind this model were to be used at European level, the required calibration could for example be performed by an independent entity. Another possibility would be to set the principles of calibration at European level so that manufacturers calibrate their own products. This procedure is currently used in the Cornish Fisher and Bootstrap methodologies.
- The DAV standard was first adopted by the DAV on 8 December 2017. It is updated yearly, mainly with respect to the calibration.

[2] For the interest rate models used see, for example, Brigo, D., Mercurio, F. (2006).

[3] The "Produktinformationsstelle Altersvorsorge gGmbH (PIA)" is the neutral authority commissioned by the Federal Ministry of Finance to classify the chances and risks of subsidised pension products. In addition, the PIA specifies the calculation methodology for the RIY listed in the national information requirements. See also: https://produktinformationsstelle.de/

[4] The basic model for capital market simulation was retrieved from the website of the PIA in July 2017.

[5] Morgen & Morgen (2013): Was available at http://www.volatium.de/fileadmin/Volatium/07_2013_Dokumentation_Volatium_Stand_ohne_pi__fuer_Volatium_Website.pdf.

2. Asset model

2.1. General remarks

The DAV standard is based on stochastic modelling of equity and interest rates (so-called basic processes). The underlying models used build upon acknowledged and commonly used models in financial mathematics. Within this framework, further assets – equity, guarantee, bond, mixed funds, money market and the conventional general account for products with profit participation – are defined and/or modelled. The approach used is based on an approach initially originated by PIA for deriving risk/reward classes of subsidised German pension products.

2.2. Basic processes / capital market model

The basic processes of interest and equity modelling are described in PIA (2017). Unfortunately, the article is not publically available. The methodology is individually provided to manufacturers that apply it for the PRIIPs calculation. The details can be also found in Brigo and Mercurio (2006). The relevant details are, however, described in the following.

2.2.1. The interest rate model

For the modelling of interest rates a so-called Two-Additive-Factor Gaussian Model (G2++ model) is applied. This kind of model allows for a sufficient meaningful sample of term structures with various levels, slopes and curvatures. In the approach the underlying short rate is at first specified assuming a risk-neutral setting and then transformed to the real-world probability measure by additionally considering some risk premiums.

The risk-neutral short rate process $r^{\star}(t)$ is specified as

$$r^*(t) = x(t) + y(t) + \psi(t), \quad r^*(0) = r_0,$$

where the processes $x(t)$ and $y(t)$ satisfy the following stochastic dynamics

$$dx(t) = -a\, x(t)dt + \sigma\, dW_1(t), \quad x(0) = 0$$
$$dy(t) = -b\, y(t)dt + \eta\, dW_2(t), \quad y(0) = 0$$

where the Wiener processes ($W_1(t)$, $W_2(t)$) have instantaneous correlation ρ and a, b, σ and η are positive constants to be calibrated each year by PIA in order to realign the short rate to current market conditions.

The deterministic function $\psi(t)$ is given by

$$\psi(t) = f^M(0,t) + \frac{\sigma^2}{2a^2}(1-e^{-at})^2 + \frac{\eta^2}{2b^2}\left(1-e^{-bt}\right)^2 + \rho \cdot \frac{\sigma\eta}{ab}(1-e^{-at})\left(1-e^{-bt}\right)$$

where $f^M(0,t)$ is the initial instantaneous forward rate for a maturity t.

As already indicated above a change of measure is performed to the real-world probability measure by adding risk premiums d_x and d_y:

$$d\tilde{x}(t) = a\,(d_x - \tilde{x}(t))dt + \sigma\, dW_1(t), \quad \tilde{x}(0) = 0$$
$$d\tilde{y}(t) = b\,(d_y - \tilde{y}(t))dt + \eta\, dW_2(t), \quad \tilde{y}(0) = 0$$

which finally yields the real-world short rate $r(t)$ as

$$r(t) = r^*(t) + d_x \cdot (1 - e^{-at}) + d_y \cdot \left(1 - e^{-bt}\right)$$

Given these specifications the price of a zero-coupon bond $P(t,T)$ at time t with maturity T can be derived by no-arbitrage arguments (cf. Brigo and Mercurio, 2006).

It is reasonable to add these risk premiums as German category 4 IBIPs are very long term products with holding periods of more than 20 years. Hence taking into regard further information via these parameters adds additional quality to this model.

2.2.2. The equity model ("basic process")

Equities are assumed to follow a (generalised) Geometric Brownian motion. Given the risk premium λ and the volatility σ_S , the equity spot price $S(t)$ hence admits the dynamics

$$dS(t) = S(t) \cdot \left((r(t) + \lambda)dt + \sigma_S\, dW(t)\right), \;\; S(0) = S_0,$$

where the Wiener process $W(t)$ is independent of $W_1(t)$ and $W_2(t)$ and λ and σ_S are recalibrated each year by PIA.

Thus, we obtain

$$S(t) = S_0 \exp\left(\int_0^t r(u)du + \left(\lambda - \frac{1}{2}\sigma_S^2\right)t + \sigma_S\, dW(t)\right)$$

The price of any other equity investment $F(t)$ with different volatility σ is modelled accordingly and is described in Section 2.3.1 in more detail.

2.3. Modelling of various types of assets

In the following we illustrate how the different asset groups can be approximated using the two basic processes for the interest rate and the equities. Here, we concentrate on main assets: equity funds, guarantee funds, bonds, money market funds, and equities. Furthermore, we show briefly how the return of a general account can be calculated. Therefore, other suggestions have to be regarded as preliminary ideas on the way forward.

2.3.1. Equity funds

For pure equity funds with volatility σ the following formula is applied:

$$F(t) = F(0) \cdot \exp\left(\int_0^t r(s)ds + \left(\lambda\frac{\sigma}{\sigma_S} - \frac{\sigma^2}{2} - K_f\right) \cdot t + \sigma W(t)\right)$$

The fund costs K_f calculated according to the RTS methodology are considered as an additional reduction of the return.

The short rate $r(s)$ is directly obtained from the capital market model.

- All other parameters only depend on the volatility σ of the equity portfolio or of the fund.

- λ represents the constant excess risk premium over the short rate and σ_s the volatility of the basic process of the equity modelling.
- Within the equity portfolio, corresponding costs K_f like ongoing charges, portfolio transaction costs can be considered as an additional reduction of the return.
- $W(t)$ denotes the Wiener process of the basic process of the equity modelling.

2.3.2. Guarantee funds

If the fund for example provides a monthly capital guarantee of e.g. 80% and this is done using CPPI, at monthly simulation intervals the CPPI algorithm for the capital preservation fund is not explicitly simulated. However, all fluctuations resulting in a loss in value of more than 20% in one month are capped at a 20% loss in value.

If the insurance product allows daily rebalancing, the CPPI algorithm with daily time step should be used.

Note: Other mechanisms for capital preservation are also possible. These should then be modelled accordingly.

If, for example, one considers a monthly capital preservation fund with an 80% security level on, e.g., an equity fund $A(t)$ with a volatility σ, then the development of the capital preservation fund $WSF(t)$ is as follows:

$$WSF(t) = WSF(t - \Delta t) \cdot \max\left\{80\%, \frac{A(t)}{A(t-\Delta t)}\right\},$$

where

$$A(t) = A(0) \cdot \exp\left(\int_0^t r(s)ds + \left(\lambda\frac{\sigma}{\sigma_S} - \frac{\sigma^2}{2} - K_f\right) \cdot t + \sigma W(t)\right)$$

and $\Delta \mathrm{t} = \frac{1}{12}$ applies.

In accordance with the RTS, the fund costs K_f are applied as an additional reduction and include both the costs of the capital preservation fund and its underlying assets.

2.3.3. Bond funds with a duration d

The performance of a bond fund $R(t)$ with duration d can be modelled as

$$R(t + \Delta t) = R(t) \cdot \frac{P(t + \Delta t, t + d)}{P(t, t + d)} \exp(-K_f \cdot \Delta t),$$

where, as above, K_f denotes the fund costs and $P(t, t + j)$ the price of a zero-coupon bond at time t with remaining time to maturity j.

2.3.4. Balanced funds consisting of equity funds with a volatility σ_{share} in the equity component and bond funds with a duration d

A mixed fund $M(t)$ with an equity portion π is expressed as

$$M(t + \Delta t) = \pi \cdot M(t) \cdot \frac{F(t + \Delta t)}{F(t)} + (1 - \pi) \cdot M(t) \cdot \frac{R(t + \Delta t)}{R(t)}$$

Here, too, the costs of the hybrid fund are to be charged as long as they have not yet been directly considered in the underlying equity or bond component.

If the volatility σ_{share} of the equity component is not available and only the volatility of the entire fund σ_{fund} is provided by the fund management company then σ_{share} can be approximated as follows:

- For duration d realisations of the bond component $R(t)$ are generated. From these realisations the volatility σ_{bond} of the bond component is estimated.
- The volatility of the equity component can then be approximated using the formula

$$\sigma_{share} \approx \sqrt{\frac{\sigma_{fund}^2 - (1-\pi)^2 \cdot \sigma_{bond}^2}{\pi^2}}$$

  It thus corresponds to a conservative estimate if the bond component is ignored. For reasons of simplification one can thus also use

$$\sigma_{share} \approx \frac{\sigma_{fund}}{\pi}$$

Alternatively the hybrid fund can, in the event of a small bond component, be modelled in a similar way to 2.3.1, though σ then represents the volatility of the entire hybrid fund and not only that of the equity component.

2.3.5. Money market funds

Money market funds can for example either be modelled as bond funds with a maximum duration of 1 or as equity funds with a volatility of close to zero or equal to zero. However, other approximations are possible.

2.3.6. Modelling the traditional general account

The traditional account is a collective investment which covers the benefits of all with profit policies. The return of this fund underlies rules of local German GAAP and can essentially be modelled as a smoothed return on equity returns and average coupon rates.

2.3.6.1. Average coupon of the portfolio of fixed-interest securities

In order to calculate the return of the general account, first of all an average coupon $R_{B,d}(t)$ is determined using the following formula:

$$R_{B,d}(t) = \frac{1}{2d} \sum_{i=1}^{2d} K(t-i).$$

- $K(t-i)$ corresponds to the swap rate at the time $t-i$ on the basis of the formula

$$K(t) = \frac{1 - P(t, t+2d)}{\sum_{j=1}^{2d} P(t, t+j)}$$

where $P(t, t+j)$ is the price of a zero coupon bond (zero bond) at the time t with time to maturity j.

- The duration d (Macaulay duration) is company-specific.
- Swap rates with non-positive time index are realised values that do not come from the simulation.

2.3.6.2. Equity

For equities or equity funds the formula of 2.3.1 is applied:

$$F(t) = F(0) \cdot \exp\left(\int_0^t r(s)ds + \left(\lambda \frac{\sigma}{\sigma_S} - \frac{\sigma^2}{2} - K_f\right) \cdot t + \sigma W(t)\right).$$

2.3.6.3. Other asset classes

If other asset classes in addition to bonds or equities have to be modelled (e.g., real estate), this can be done by means of a risk-adequate weighted allocation of this asset class to bonds and equities. Accordingly, the volatility of the other asset classes is the weighted average of the bond and equity volatility.

2.3.6.4. Return on the general account

To determine the return of the general account a weighted average is modelled from the average coupon of fixed-interest securities and the realised equity return using the following formula:

$$R(t) = \sqrt[3]{\prod_{i=0}^{2}\left(\Psi \cdot \frac{F(t-i)}{F(t-i-1)} + (1-\Psi) \cdot \left(1 + R_{B,d}(t-i)\right)\right)} - 1.$$

The following values are specific to the company:

- duration d
- volatility of the equity component σ
- share of equities in the general account Ψ

Similar to the treatment of negative time indices for swap rates (cf. Section 2.3.6.1), the realised returns of the equity portfolio are used for $F(t-i)$ with negative time index $t-i$.

2.3.6.5. Profit participation / Total return allocated to policyholders

The return $R(t)$ on the general account is used to model the total return $g(t)$ allocated to the policyholder after profit participation. Any costs or fees with effect on the return to the policyholder have to be deducted in accordance with the national provisions and the provision made in the RTS.

Considering this the total return is calculated as follows:

$$g(t+1) = H(t) + \left((R(t) - K - K' - E) - H(t)\right)^+.$$

where:

- $H(t)$ is the contract-specific guaranteed return in year t.

- K is the company-specific cost for managing the investments in relation to the average investments at the market values of the last financial year. It can be deduced from the financial statements.
- K' represents the not yet considered portfolio transaction costs.

  Note that both the portfolio transaction costs of the fixed-interest investments and of the equity component have to be considered. Moreover, it should be ensured that, in connection with the modelling of the equity component (cf. Section 2.3.6.2), the portfolio transaction costs are not counted twice.

- E denotes the best-estimate participation of the shareholder. Legal thresholds due to (local) supervisory law have to be considered:
  - point 34 (c) of Annex IV of the RTS: This point states that the assumptions governing the profit sharing between insurer and policyholder shall be in line with the current business practice and business strategy of the PRIIP manufacturer. For life insurers within the scope of Directive 2009/138/EC, these assumptions shall be consistent with the assumptions on future management actions used for the valuation of technical provisions in the Solvency II balance sheet. One-off changes in the participation rate compared to previous years may therefore be considered by averaging with the rates of previous years.
  - point 34 (d) of Annex IV of the RTS: This point states that where a component of the performance relates to profit participation that is “payable on a discretionary basis”, this component shall only be assumed in the favourable performance scenario. This includes products whose profit participation is not, or very little, subject to statutory regulation. This limitation does not apply to German regulations governing profit participation.

3. Calibration of the asset model

As previously mentioned, the DAV standard for category 4 PRIIPs uses the calibration provided by the PIA in order to achieve a uniform calibration. The calibration is updated on a yearly basis, which ensures the estimation of the model parameters under current market conditions by an independent institution. As the calibration is licensed by PIA, it is not possible to display the actual values of parameters in this paper.

Parameters for basic processes to be provided		
interest rate model, initial interest rate curve	initial interest rate curve	$f^M(0,t)$
interest rate model, parameters of G2++	mean reversion speed of x(t)	a
	volatility of x(t)	σ
	mean reversion speed of y(t)	b
	volatility of y(t)	η
	correlation between x(t) and y(t)	ρ
interest rate model, risk premium	risk premium on short rate for process x(t)	d_x
	risk premium on short rate for process y(t)	d_y
equity model	constant risk premium on the short rate	λ
	volatility of the basic process of the equity modelling	σ_S

However, if the necessary guidance is given, the calibration of the model can be easily performed by each undertaking itself. For example, the parameters of the G2++ interest rate model are calibrated to observed interest-rate cap, floor or swaption data. Further explanations can be found in Brigo, D., Mercurio, F., 2006[6].

Depending on the individual product the following parameters are needed. So far they have been provided within the European PRIIPs Template (EPT) and the Comfort European PRIIPs Template (CEPT) by the fund industry[7]. The templates are broadly used by fund managers and insurers in Germany.

Individual parameters for each relevant asset / fund provided by EPT / CEPT or estimated based on market values of the past 5 years	
volatility of the equity portfolio or of the fund	σ
costs of the fund	K_f
duration of the fund	d

[6] Brigo, D., Mercurio, F., S. 166 ff.

[7] The templates can be found on the FinDatEx website: https://findatex.eu/

References

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Legal Sources

PRIIPs Regulation	Regulation (EU) No 1286/2014 of The European Parliament and the Council of 26 November 2014 on key information documents for packaged retail and insurance-based investment products (PRIIPs) http://eur-lex.europa.eu/legal-content/DE/TXT/HTML/?uri=OJ:JOL_2014_352_R_0001&from=EN
Delegated EU-Regulation / Regulatory Technical Standard / RTS	Commission delegated Regulation (EU) 2017/653 of 8 March 2016; http://eur-lex.europa.eu/legal-content/DE/TXT/PDF/?uri=CELEX:32017R0653&from=EN
Pension Provision Agreements Certification Act (in German)	http://www.gesetze-im-internet.de/altzertg/index.html
Life Insurance Reform Act (in German)	German Federal Law Gazette 2014 Part I No. 38, pp. 1330-1337; http://www.bgbl.de/xaver/bgbl/start.xav?startbk=Bundesanzeiger_BGBl&jumpTo=bgbl114s1330.pdf